MHT304 - Plan du cours

1. Chapitre introductif ;
2. Résolution numérique des équations ;
3. Polynômes ; élimination, interpolation, approximation ;
4. Initiation aux méthodes itératives ;
5. Calcul numérique et équations différentielles.

`Notes de cours en ligne :`
`http:\\www.math.u-bordeaux1.fr\~yger\mht304.pdf`
`Contrat sous Ulysse prevu`

Une référence :
Mathématiques Appliquées L3, A. Yger et J.A. Weil, Pearson Education, 2009.

La force du calcul symbolique.

$$\pi = 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

Sous MAPLE 10**:**

```
> f:=1:
  for i from 1 to 200 do
  f:=f+(-1)^(i)/(2*i+1) ;
  end do:
> f;
7352288428557188872088197148101204\
3066665106521456350001612397888386\
3531933421116083310014790530785004\
7293430335872417675527229281927997\
7773557277737525954598578338645703\
821/934642317507958856584804653282\
3095100905700257562749503796198101\
1259232103152868300177281736010020\
0684169771518249599308707249123032\
2985370303032641837928861710357341\
2561125
> evalf(4*f,50);
3.14656774718295640128778766018983\
24180868624240507
```

$$\pi = 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

Sous MAPLE 10**:**

```
> f:=1:
  for i from 1 to 199 do
  f:=f+(-1)^(i)/(2*i+1) ;
  end do:
> f;
1827675970097420506801275495709454\
4386368026761356191103045758155876\
9338489356620211569295659630689228\
4328435674841787983158902277238825\
7059031621722706655123670132787429\
6/23307788466532639815082410306292\
0077329319208418023678398907683319\
8484591101069034917139195411721198\
7136403279756847863060031150200306\
6966840474629472267552661106168111\
125
> evalf(4*f,50);
3.13659268483881675041496970507761\
29667152913517315
```

$$\pi = 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\Big(4(1/5)^{2k+1} - (1/239)^{2k+1}\Big)$$

Sous MAPLE 10 **:**

```
> f:=4/5-1/239:
  for i from 1 to 199 do
  f:=f+((-1)^(i)/(2*i+1))
    *(4*(1/5)^(2*i+1)-(1/239)^(2*i+1)) ;
  end do:
> evalf(4*f,50);
3.14159265358979323846264338327950\
28841971693993751
```

```
> f:=4/5-1/239:
  for i from 1 to 200 do
  f:=f+((-1)^(i)/(2*i+1))
    *(4*(1/5)^(2*i+1)-(1/239)^(2*i+1)) ;
  end do:
> evalf(4*f,50);
3.14159265358979323846264338327950\
28841971693993751
```

À propos de stabilité (un exemple troublant).

Sous MATLAB 7 **:**

```
>> M= [10 7 8 7;7 5 6 5;8 6 10 9;7 5 9 10];
>> M
M =
    10     7     8     7
     7     5     6     5
     8     6    10     9
     7     5     9    10

>> B = [32;23;33;31];
>> B
B =
    32
    23
    33
    31

>> M^(-1)*B
ans =

    1.0000
    1.0000
    1.0000
    1.0000
```

Sous MATLAB 7 **:**

```
>> perturb=(1/100)*(rand(4,4)-ones(4,4)/2);
>> perturb

perturb =

   -0.0005    0.0035    0.0034    0.0033
    0.0043    0.0003   -0.0048    0.0000
   -0.0003   -0.0030    0.0018    0.0021
   -0.0008    0.0017   -0.0012   -0.0007

>> Mperturb=M+perturb;
>> Mperturb^(-1)*B

ans =

    0.6926
    1.5043
    0.8775
    1.0733
```

Sous MATLAB 7 :

```
>> M
M   =
    10     7     8     7
     7     5     6     5
     8     6    10     9
     7     5     9    10

>> M^(-1)
ans =

   25.0000  -41.0000   10.0000   -6.0000
  -41.0000   68.0000  -17.0000   10.0000
   10.0000  -17.0000    5.0000   -3.0000
   -6.0000   10.0000   -3.0000    2.0000
```

Résoudre sous MAPLE : le calcul symbolique

$$x^5 + \frac{x^3}{2} + 1 = 0$$

```
> solve (x^5 + x^3/2 +1 =0);
{x = RootOf(2*_Z^5+_Z^3+2, index = 1)},
{x = RootOf(2*_Z^5+_Z^3+2, index = 2)},
{x = RootOf(2*_Z^5+_Z^3+2, index = 3)},
{x = RootOf(2*_Z^5+_Z^3+2, index = 4)},
{x = RootOf(2*_Z^5+_Z^3+2, index = 5)}

> solve (x^3 + x^2/2 + x + 1 =0);
{x = -1/6*(91+6*sqrt(267))^(1/3)
     +11/6/(91+6*sqrt(267))^(1/3)-1/6},
{x = 1/12*(91+6*sqrt(267))^(1/3)
     -11/12/(91+6*sqrt(267))^(1/3)-1/6
+1/2*I*sqrt(3)*(-1/6*(91+6*sqrt(267))^(1/3)
     -11/6/(91+6*sqrt(267))^(1/3))},
{x = 1/12*(91+6*sqrt(267))^(1/3)
     -11/12/(91+6*sqrt(267))^(1/3)-1/6
-1/2*I*sqrt(3)*(-1/6*(91+6*sqrt(267))^(1/3)
     -11/6/(91+6*sqrt(267))^(1/3))}

> fsolve(x^5 + x^3/2 + 1 =0);
-.9098248906
```

Les méthodes : Newton [MATLAB 7]

$$x_{k+1} = x_k - \frac{x_k^5 + x_k^3/2 + 1}{5x_k^4 + 3x_k^2/2} = x_k - \frac{f(x_k)}{f'(x_k)}\,.$$

```
function x=Newton2(init,N);
x=init;
for i=1:N
    x = x - (x^5+x^3/2+1)/(5*x^4+3*x^2/2);
end
```

```
>> Newton2(-1,3)
  -0.909825093948150
>> Newton2(-1,5)
  -0.909824890637916
>> Newton2(-1,10)
  -0.909824890637916
```

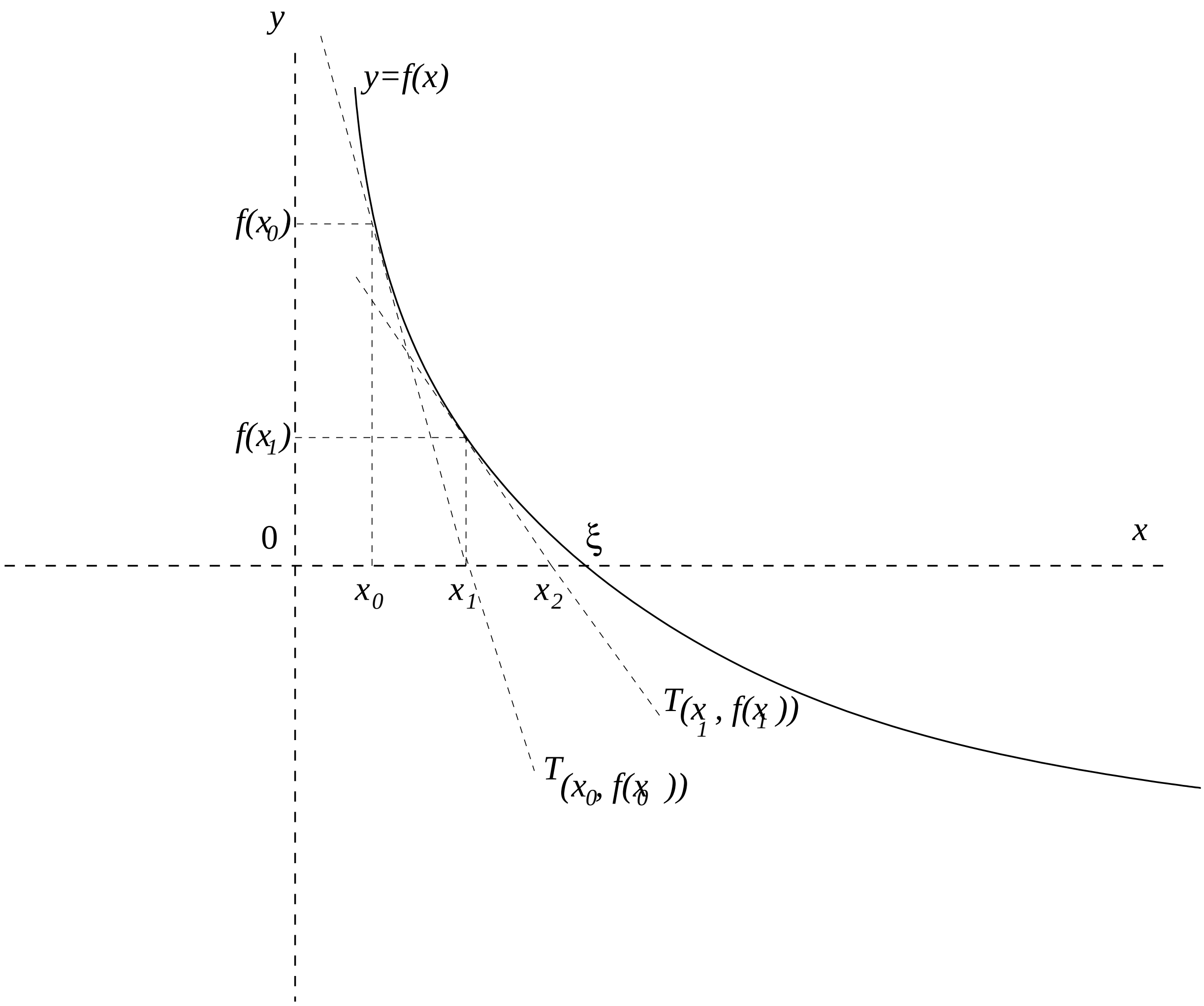

Figure 1: Méthode de Newton