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TIME-INDEXED FORMULATIONS OF THE TRUCK-TO-DOOR SCHEDULING PROBLEM AT MULTI-DOOR CROSS-DOCKING TERMINALS WITH TEMPORARY STORAGE

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ABSTRACT

A cross-docking terminal is a transshipment facility in supply chains, where products transported by inbound trucks are unloaded at inbound doors, sorted, and reloaded on outbound trucks at outbound doors. In this study we address the truck-to-door scheduling problem in a multi-door cross-docking terminal where temporary storage is considered. We propose two types of time-indexed formulation for the problem to assign trucks to dock doors and determine their arrival and departure times so that tardiness and earliness as well as unsatisfied demand are minimized. We examine the effectiveness of the proposed formulations by numerical experiment.

NOMENCLATURE

C: the capacity of the cross-docking terminal.
I: the number of inbound doors.
M: the number of inbound trucks.
N: the number of outbound trucks.
O: the number of outbound doors.
P: the number of product types.
T: the length of the planning horizon.
$\mathbb{Z}^+$: the set of nonnegative integers.
a_{pm}: the number of units of product type p supplied by inbound truck m.
b_{pn}: the number of units of product type p requested from outbound truck n.
c_n: the unit penalty for the earliness and tardiness of outbound truck n.
d_n: the duedate of outbound truck n.
f_{nt}: the earliness and tardiness penalties for outbound truck n to leave at time t: $f_{nt} := c_n \max(t-d_n, d_n-t)$.
l_n: the release time (earliest arrival time) of outbound truck n.
p_p: the unit penalty for the unsatisfied demand for product type p.
r_m: the release time of inbound truck m.
t_{io}: the transfer time from inbound door i to outbound door o.

1 INTRODUCTION

Cross-docking terminals play an important role in supply chains for reducing inventory, storage, and handling costs. An inbound truck that arrives at a cross-docking terminal is assigned to an inbound door, where it unloads products. These products are sorted and transferred across the dock to outbound doors where they are loaded on outbound trucks. Various types of cross-docking problem have been studied so far [1, 2]. However, most studies assume that products are transferred between doors directly and do not consider intermediate storage inside the dock. In this study we will address the cross-docking problem for multi-door cross-docking terminals to assign trucks to dock doors and determine their arrival and departure times with temporary storage being considered. Following [2], we refer to this type of problem, i.e. the problem of determining both truck-to-door assignment and truck arrival/departure times, as the truck-to-door scheduling problem.

In the cross-docking problem in [3, 4], the capacity of the cross-docking terminal is taken into account, whereas the arrival and departure times of trucks are fixed. Sadykov [5] studied the problem of determining arrival and departure times of trucks so that the storage cost is minimized, where only one inbound door and one outbound door are considered. Ladier and Alpan [6] addressed a truck scheduling problem under soft time-window constraints on truck arrival/departure times. In their problem setting, truck-to-door assignment is not considered, although temporary storage is assumed. Assadi and Bagheri [7] gave an ILP formulation of a truck-to-door scheduling problem to minimize earliness and tardiness penalties where temporary storage buffers are assumed to be located at every inbound door.

In this study, we propose two types of ILP formulation for a similar problem to that in [7]. The primary differences from [7] are:

1. We consider the capacity of the cross-dock terminal.
2. Unloading and loading times of products, and truck changeover time in the formulation in [7] are ignored in our model.
3. We give time-indexed formulations, unlike the formulation in [7] where arrival and departure times of trucks are part of decision variables.

We will examine the effectiveness of the proposed formulations by numerical experiment.

2 PROBLEM DESCRIPTION

Consider that P types of products are transported from M origins to N destinations via a cross-docking terminal with I inbound doors and O outbound doors. A total of a_{pm} units of product type p is transported from origin m. Similarly, b_{pn} units of product type p is transported to destination n. An inbound truck m transports products from origin m and stays at a inbound door, where it unloads them. The arrival time of inbound truck m should not be earlier than r_m. Material handling equipment such as forklifts transfers products from inbound doors to outbound doors. An outbound truck n loads products at a outbound door and transports them to destination n. The earliest arrival time and duedate of outbound truck n are l_n and d_n, respectively. Preemption of truck-to-door assignment is not allowed: A truck cannot return to a door again once it leaves a door. The time required for transferring products from inbound door i to outbound door o is given by t_{io}. The products that cannot be transferred directly from inbound doors to outbound doors can be kept temporarily in storage buffers at individual inbound doors. If demand for product type p is not satisfied, a penalty p_p is incurred for each unit of the unsatisfied demand. The capacity of the cross-docking terminal in each period is given by C, and the number of units transferred and being transferred plus the number of units kept in storage buffers should not exceed C in each period. All these parameters are assumed to be integers. The objective is to find an optimal assignment of trucks to doors in the planning horizon $[0,T)$ that minimizes the earliness and tardiness penalties plus the total penalty for unsatisfied demand.

3 PROBLEM FORMULATION

In this section we give an ILP formulation of our problem. The formulation uses the following binary decision variables:

- x^{I}_{mist}: 1 iff inbound truck m is assigned to inbound door i in duration $[s,t)$. $1 \le m \le M$, $1 \le i \le I$, $r_m \le s < t \le T$.
- x^{O}_{nost}: 1 iff outbound truck n is assigned to outbound door o in duration $[s,t)$. $1 \le n \le N$, $1 \le o \le O$, $l_n \le s < t \le T$.

Integer decision variables are summarized as follows:

- g^{I}_{pmit}: The number of units of product type p transferred from inbound truck m at inbound door i to outbound doors in duration $[t,t+1)$. $1 \le p \le P$, $1 \le m \le M$, $1 \le i \le I$, $r_m \le t \le T-1$.
- h^{I}_{pmit}: The number of units of product type p transferred to storage buffer i from inbound truck m in duration $[t,t+1)$. $1 \le p \le P$, $1 \le m \le M$, $1 \le i \le I$, $r_m \le t \le T-1$.
- g^{O}_{pnot}: The number of units of product type p transferred to outbound truck n at outbound door o in duration $[t,t+1)$. $1 \le p \le P$, $1 \le n \le N$, $1 \le o \le O$, $l_n \le t \le T-1$. For ease of notation, we assume that $g^{\mathrm{O}}_{pnot} = 0$ for $t < l_n$.
- h^{O}_{pit}: The number of units of product type p transferred from storage buffer i in duration $[t,t+1)$. $1 \le p \le P$, $1 \le i \le I$, $0 \le t \le T-1$.
- q_{piot}: The number of units of product type p transferred from inbound door i to outbound door o in duration $[t,t+t_{io}+1)$. $1 \le p \le P$, $1 \le i \le I$, , $1 \le o \le O$, $0 \le t \le T-t_{io}-1$. Let $q_{piot} = 0$ if $t<0$ or $t \ge T-t_{io}$ for ease of notation.
- s_{pit}: The number of units of product type p stored in storage buffer i at time t. $1 \le p \le P$, $1 \le i \le I$, $1 \le t \le T-1$, and $s_{pi0} = s_{piT} = 0$.

Introducing the earliness-tardiness coefficient f_{nt} by $f_{nt} = c_n \max(t-d_n, d_n-t)$, we obtain the ILP (P1):

$$\min \sum_{n=1}^{N} \sum_{t=l_n+1}^{T} f_{nt} \sum_{o=1}^{O} \sum_{s=l_n}^{t-1} x^{\mathrm{O}}_{nost} + \sum_{p=1}^{P} p_p \sum_{n=1}^{N} \left(b_{pn} - \sum_{o=1}^{O} \sum_{t=0}^{T-1} g^{\mathrm{O}}_{pnot} \right), \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{I} \sum_{s=r_m}^{T-1} \sum_{t=s+1}^{T} x^{\mathrm{I}}_{mist} = 1, \quad 1 \le m \le M, \tag{2}$$

$$\sum_{o=1}^{O} \sum_{s=l_n}^{T-1} \sum_{t=s+1}^{T} x^{\mathrm{O}}_{nost} = 1, \quad 1 \le n \le N, \tag{3}$$

$$\sum_{m=1}^{M} \sum_{s=r_m}^{t} \sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu} \le 1, \quad 1 \le i \le I,\ 0 \le t \le T-1, \tag{4}$$

$$\sum_{n=1}^{N}\sum_{s=l_n}^{t}\sum_{u=t+1}^{T} x^{\mathrm{O}}_{nosu} \le 1, \quad 1 \le o \le O,\ 0 \le t \le T-1, \tag{5}$$

$$\sum_{t=r_m}^{T-1} (g^{\mathrm{I}}_{pmit} + h^{\mathrm{I}}_{pmit}) \le a_{pm} \sum_{s=r_m}^{T-1}\sum_{t=s+1}^{T} x^{\mathrm{I}}_{mist}, \quad 1 \le p \le P,\ 1 \le m \le M,\ 1 \le i \le I, \tag{6}$$

$$g^{\mathrm{I}}_{pmit} + h^{\mathrm{I}}_{pmit} \le a_{pm} \sum_{s=r_m}^{t}\sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu}, \quad 1 \le p \le P,\ 1 \le m \le M,\ 1 \le i \le I,\ r_m \le t \le T-1, \tag{7}$$

$$\sum_{t=l_n}^{T-1} g^{\mathrm{O}}_{pnot} \le b_{pn} \sum_{s=l_n}^{T-1}\sum_{t=s+1}^{T} x^{\mathrm{O}}_{nost}, \quad 1 \le p \le P,\ 1 \le n \le N,\ 1 \le o \le O, \tag{8}$$

$$g^{\mathrm{O}}_{pnot} \le b_{pn} \sum_{s=l_n}^{t}\sum_{u=t+1}^{T} x^{\mathrm{O}}_{nosu}, \quad 1 \le p \le P,\ 1 \le n \le N,\ 1 \le o \le O,\ l_n \le t \le T-1, \tag{9}$$

$$\sum_{m=1}^{M} g^{\mathrm{I}}_{pmit} + h^{\mathrm{O}}_{pit} = \sum_{o=1}^{O} q_{piot}, \quad 1 \le p \le P,\ 1 \le i \le I,\ 0 \le t \le T-1, \tag{10}$$

$$\sum_{i=1}^{I} q_{pio,t-t_{io}} = \sum_{n=1}^{N} g^{\mathrm{O}}_{pnot}, \quad 1 \le p \le P,\ 1 \le o \le O,\ 0 \le t \le T-1, \tag{11}$$

$$s_{pit} = s_{pi,t-1} - h^{\mathrm{O}}_{pit} + \sum_{m=1}^{M} h^{\mathrm{I}}_{pmit}, \quad 1 \le p \le P,\ 1 \le i \le I,\ 1 \le t \le T, \tag{12}$$

$$\sum_{p=1}^{P}\sum_{i=1}^{I}\left(s_{pi,t+1} + \sum_{o=1}^{O}\sum_{s=t-t_{io}}^{t} q_{pios} \right) \le C, \quad 0 \le t \le T-1, \tag{13}$$

$$g^{\mathrm{I}}_{pmit} \in \mathbb{Z}^+, \quad 1 \le p \le P,\ 1 \le m \le M,\ 1 \le i \le I,\ r_m \le t \le T-1, \tag{14}$$

$$g^{\mathrm{O}}_{pnot} \in \mathbb{Z}^+, \quad 1 \le p \le P,\ 1 \le n \le N,\ 1 \le o \le O,\ l_n \le t \le T-1, \tag{15}$$

$$h^{\mathrm{I}}_{pmit} \in \mathbb{Z}^+, \quad 1 \le p \le P,\ 1 \le m \le M,\ 1 \le i \le I,\ r_m \le t \le T-1, \tag{16}$$

$$h^{\mathrm{O}}_{pit} \in \mathbb{Z}^+, \quad 1 \le p \le P,\ 1 \le i \le I,\ 0 \le t \le T-1, \tag{17}$$

$$q_{piot} \in \mathbb{Z}^+, \quad 1 \le p \le P,\ 1 \le i \le I,\ 1 \le o \le O,\ 0 \le t \le T-1, \tag{18}$$

$$s_{pit} \in \mathbb{Z}^+, \quad 1 \le p \le P,\ 1 \le i \le I,\ 0 \le t \le T, \tag{19}$$

$$x^{\mathrm{I}}_{mist} \in \{0, 1\}, \quad 1 \le m \le M,\ 1 \le i \le I,\ r_m \le s < t \le T, \tag{20}$$

$$x^{\mathrm{O}}_{nost} \in \{0, 1\}, \quad 1 \le n \le N,\ 1 \le o \le O,\ l_n \le s < t \le T. \tag{21}$$

The first term of the objective function (1) of (P1) describes the earliness and tardiness penalties, and the second term is the penalty for unsatisfied demand. Constraints (2) ensure that each inbound truck i is assigned to an inbound door exactly once. Similarly, constraints (3) ensure that each outbound truck o is assigned to an outbound door exactly once. Constraints (4) and (5) guarantee that each door is occupied by at most one truck in each duration $[t, t+1)$ ((4) for inbound door i and (5) for outbound door o.) From constraints (6), the total number of units of product type p unloaded from inbound truck m at inbound door i is at most a_{pm} if inbound truck m is assigned to inbound door i, and equal to 0 otherwise. In duration $[t, t+1)$, the number of units of product type p transferred from inbound truck m at inbound door i is at most a_{pm} if inbound truck m is at inbound door i in this duration, and equal to 0 otherwise. It is ensured by constraints (7). Constraints (8) and (9) are counterparts of (6) and (7), respectively, for outbound truck n and outbound door o. Constraints (10) guarantee that the total number of units of product type p transferred from inbound door i in the duration starting at t is equal to the total number of units of product type p transferred from inbound door i and storage buffer i in duration $[t, t+1)$. Similarly, constraints (11) guarantee that the total number of units of product type p transferred from inbound door i in the duration starting at $t - t_{io}$ and thus ending at $t+1$ is equal to the number of units of product type p transferred to outbound door o in duration $[t, t+1)$. Constraints (12) define the increase of product type p stored in storage buffer i from time $t-1$ to t: The second and third terms of the righthand side denote the numbers of units transferred from storage buffer i and to storage buffer i, respectively. Finally, constraints (13) ensure that the total number of units of products being transferred and stored in duration $[t, t+1)$ does not exceed capacity C.

4 COMPACT FORMULATION

In the formulation (P1) in the preceding section, we assigned a binary decision variable to every pair of arrival and departure times of a truck as in [6]. Thus the number of binary decision variables is approximately $(MI + NJ)T(T+1)$. In this section, we provide an alternative formulation with at most $2(MI + NJ)T$ binary decision variables.

This formulation is inspired by formulations of the unit commitment problem in the literature [8–10]. The objective of the unit commitment problem is to determine the daily on-off schedule of (thermal) power generators so as to minimize operational cost. In the majority of the formulations of the unit commitment problem, the state (on or off) of a generator in each time slot is expressed by a binary decision variable (state binary decision variable). To further model the startup cost, startup binary decision variables are introduced that take one iff the generator is turned on in the corresponding time slot. The shutdown cost is modeled in a similar way using shutdown binary decision variables.

In the new formulation (P2), we introduce the following binary decision variables for trucks in place of x^{I}_{mist} and x^{O}_{nost} in (P1):

y^{I}_{mit}: 1 iff inbound truck m departs from inbound door i at t. $1 \le m \le M$, $1 \le i \le I$, $r_m + 1 \le t \le T$.

u^{I}_{mit}: 1 iff inbound truck m occupies inbound door i in duration $[t, t+1)$. $1 \le m \le M$, $1 \le i \le I$, $r_m \le t \le T-1$. We assume $u^{\mathrm{I}}_{mit} = 0$ for $t < r_m$ and $t = T$.

y^{O}_{not}: 1 iff outbound truck n departs from outbound door o at t. $1 \le n \le N$, $1 \le o \le O$, $l_n + 1 \le t \le T$.

u^{O}_{not}: 1 iff outbound truck n occupies outbound door o in duration $[t, t+1)$. $1 \le n \le N$, $1 \le o \le O$, $l_n \le t \le T-1$. We assume $u^{\mathrm{O}}_{not} = 0$ for $t < l_n$ and $t = T$.

Using these decision variables together with integer ones g^{I}_{pmit}, h^{I}_{pmit}, g^{O}_{pnot}, h^{O}_{pit}, q_{piot}, and s_{pit}, we obtain the following formulation (P2).

$$\min \sum_{n=1}^{N} \sum_{t=l_n+1}^{T} f_{nt} \sum_{o=1}^{O} y^{\mathrm{O}}_{not} + \sum_{p=1}^{P} p_p \sum_{n=1}^{N} \left(b_{pn} - \sum_{o=1}^{O} \sum_{t=0}^{T-1} g^{\mathrm{O}}_{pnot} \right), \tag{22}$$

$$\text{s.t.} \sum_{i=1}^{I} \sum_{t=r_m+1}^{T} y^{\mathrm{I}}_{mit} = 1, \quad 1 \le m \le M, \tag{23}$$

$$\sum_{o=1}^{O} \sum_{t=l_n+1}^{T} y^{\mathrm{O}}_{not} = 1, \quad 1 \le n \le N, \tag{24}$$

$$\sum_{m=1}^{M} u^{\mathrm{I}}_{mit} \le 1, \quad 1 \le i \le I,\ 0 \le t \le T-1, \tag{25}$$

$$\sum_{n=1}^{N} u^{\mathrm{O}}_{not} \le 1, \quad 1 \le o \le O,\ 0 \le t \le T-1, \tag{26}$$

$$\sum_{t=r_m}^{T-1} (g^{\mathrm{I}}_{pmit} + h^{\mathrm{I}}_{pmit}) \le a_{pm} \sum_{t=r_m+1}^{T} y^{\mathrm{I}}_{mit}, \quad 1 \le p \le P,\ 1 \le m \le M,\ 1 \le i \le I, \tag{27}$$

$$g^{\mathrm{I}}_{pmit} + h^{\mathrm{I}}_{pmit} \le a_{pm} u^{\mathrm{I}}_{mit}, \quad 1 \le p \le P,\ 1 \le m \le M,\ 1 \le i \le I,\ r_m \le t \le T-1, \tag{28}$$

$$\sum_{t=l_n}^{T-1} g^{\mathrm{O}}_{pnot} \le b_{pn} \sum_{s=l_n}^{T-1} y^{\mathrm{O}}_{not}, \quad 1 \le p \le P,\ 1 \le n \le N,\ 1 \le o \le O, \tag{29}$$

$$g^{\mathrm{O}}_{pnot} \le b_{pn} u^{\mathrm{O}}_{not}, \quad 1 \le p \le P,\ 1 \le n \le N,\ 1 \le o \le O,\ l_n \le t \le T-1, \tag{30}$$

$$y^{\mathrm{I}}_{mit} \le u^{\mathrm{I}}_{mi,t-1}, \quad 1 \le m \le M,\ 1 \le i \le I,\ r_m + 1 \le t \le T, \tag{31}$$

$$y^{\mathrm{I}}_{mit} \ge u^{\mathrm{I}}_{mi,t-1} - u^{\mathrm{I}}_{mit}, \quad 1 \le m \le M,\ 1 \le i \le I,\ r_m + 1 \le t \le T, \tag{32}$$

$$y^{\mathrm{O}}_{not} \le u^{\mathrm{O}}_{no,t-1}, \quad 1 \le n \le N,\ 1 \le o \le O,\ l_n + 1 \le t \le T, \tag{33}$$

$$y^{\mathrm{O}}_{not} \ge u^{\mathrm{O}}_{no,t-1} - u^{\mathrm{O}}_{not}, \quad 1 \le n \le N,\ 1 \le o \le O,\ l_n + 1 \le t \le T, \tag{34}$$

(10)–(19),

$$y^{\mathrm{I}}_{mit} \in \{0, 1\},\ 1 \le m \le M,\ 1 \le i \le I,\ r_m + 1 \le t \le T, \tag{35}$$

$$u^{\mathrm{I}}_{mit} \in \{0, 1\},\ 1 \le m \le M,\ 1 \le i \le I,\ r_m \le t \le T-1, \tag{36}$$

$$y^{\mathrm{O}}_{not} \in \{0, 1\},\ 1 \le n \le N,\ 1 \le o \le O,\ l_n + 1 \le t \le T, \tag{37}$$

$$u^{\mathrm{O}}_{not} \in \{0, 1\},\ 1 \le n \le N,\ 1 \le o \le O,\ l_n \le t \le T-1, \tag{38}$$

Constraints (23)–(30) correspond to (2)–(9) in (P1), respectively. Constraints (31) guarantee that $y^{\mathrm{I}}_{mit} = 0$ if $u^{\mathrm{I}}_{mi,t-1} = 0$, that is, the departure time of inbound truck m from inbound door i is not equal to t if the truck is not at the door in duration $[t-1, t)$. Constraints (32) ensure that $y^{\mathrm{I}}_{mit} = 1$ when $u^{\mathrm{I}}_{mi,t-1} = 1$ and $u^{\mathrm{I}}_{mit} = 0$. Constraints (33) and (34) are counterparts of constraints (31) and (32), respectively, for outbound trucks.

The number of binary decision variables in (P1) is $(MI + NJ)T(T+1)$ at the maximum, while that in (P2) is at most $2(MI + NJ)T$. Nevertheless, the linear relaxation of (P2) provides the same lower bound as that of (P1), so that we can expect that (P2) is easier to solve to optimality. The following theorem explicitly describe the equivalence of the two linear relaxations.

Theorem 1. *Let (LP1) and (LP2) be the linear relaxations of (P1) and (P2), respectively. Then, the optimal objective values of (LP1) and (LP2) are the same.*

Proof. Since the decision variables (g^{I}_{pmit}, h^{I}_{pmit}, g^{O}_{pnot}, h^{O}_{pit}, q_{piot}, s_{pit}) are the same in (P1) and (P2), we will prove that a feasible solution (x^{I}_{mist}, x^{O}_{nost}) of (LP1) can be converted to a feasible solution (y^{I}_{mit}, u^{I}_{mit}, y^{O}_{not}, u^{O}_{not}) of (LP2) and vice versa, without changing the objective value.

First, we convert a feasible solution (x^{I}_{mist}, x^{O}_{nost}) of (LP1) to a feasible solution (y^{I}_{mit}, u^{I}_{mit}, y^{O}_{not}, u^{O}_{not}) of (LP2). Let us define y^{I}_{mit}, u^{I}_{mit}, y^{O}_{not}, and u^{O}_{not} as follows:

$$y^{\mathrm{I}}_{mit} = \sum_{s=r_m}^{t-1} x^{\mathrm{I}}_{mist} \tag{39}$$

$$u^{\mathrm{I}}_{mit} = \sum_{s=r_m}^{t} \sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu}, \tag{40}$$

$$y^{\mathrm{O}}_{not} = \sum_{s=l_n}^{t-1} x^{\mathrm{O}}_{nost}, \tag{41}$$

$$u^{\mathrm{O}}_{not} = \sum_{s=l_n}^{t} \sum_{u=t+1}^{T} x^{\mathrm{O}}_{nosu}. \tag{42}$$

Then, it is obvious that (1), (4)–(9) are rewritten as (22), (25)–(30), respectively. Noting

$$\sum_{s=r_m}^{T-1} \sum_{t=s+1}^{T} x^{\mathrm{I}}_{mist} = \sum_{t=r_m+1}^{T} \sum_{s=r_m}^{t-1} x^{\mathrm{I}}_{mist}, \tag{43}$$

we can also rewrite (2) as (23). A similar argument holds true for

(3) and (24). Furthermore, (31) follows from

$$u^{\mathrm{I}}_{mi,t-1} = \sum_{s=r_m}^{t-1}\sum_{u=t}^{T} x^{\mathrm{I}}_{misu} = \sum_{s=r_m}^{t-1} x^{\mathrm{I}}_{mist} + \sum_{s=r_m}^{t-1}\sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu}$$
$$= y^{\mathrm{I}}_{mit} + \sum_{s=r_m}^{t-1}\sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu} \geq y^{\mathrm{I}}_{mit}, \tag{44}$$

and (32) from

$$u^{\mathrm{I}}_{mi,t-1} - u^{\mathrm{I}}_{mit} = y^{\mathrm{I}}_{mit} + \sum_{s=r_m}^{t-1}\sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu} - u^{\mathrm{I}}_{mit}$$
$$= y^{\mathrm{I}}_{mit} + \sum_{s=r_m}^{t-1}\sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu} - \sum_{s=r_m}^{t}\sum_{u=t+1}^{T} x^{\mathrm{I}}_{misu}$$
$$= y^{\mathrm{I}}_{mit} - \sum_{u=t+1}^{T} x^{\mathrm{I}}_{mitu} \leq y^{\mathrm{I}}_{mit}. \tag{45}$$

Again, (33) and (34) can be derived similarly.

Next, we convert a feasible solution (y^{I}_{mit}, u^{I}_{mit}, y^{O}_{not}, u^{O}_{not}) of (LP2) to a feasible solution (x^{I}_{mist}, x^{O}_{nost}) of (LP1). Here, we only consider converting the decision variables for outbound doors and trucks, y^{O}_{not} and u^{O}_{not}. We apply the following procedure:

1° Let t^* be the minimum t satisfying $y^{\mathrm{O}}_{not} > 0$.

2° While $y^{\mathrm{O}}_{not^*} > 0$, repeat the following:

  (a) Let s^* be the maximum s satisfying $u^{\mathrm{O}}_{no,s-1} < u^{\mathrm{O}}_{nos}$ and $s < t^*$. Let $\delta := \min(u^{\mathrm{O}}_{nos^*} - u^{\mathrm{O}}_{no,s^*-1}, y^{\mathrm{O}}_{not^*})$.

  (b) Let $y^{\mathrm{O}}_{not^*} := y^{\mathrm{O}}_{not^*} - \delta$, $u^{\mathrm{O}}_{nou} := u^{\mathrm{O}}_{nou} - \delta$ for $s^* \leq u < t^*$, and $x^{\mathrm{O}}_{nos^*t^*} := \delta$.

3° If $y^{\mathrm{O}}_{not} = 0$ for all t, terminate. Otherwise, go to 1°.

From (34), $y^{\mathrm{O}}_{not} > 0$ if $u^{\mathrm{O}}_{no,t-1} > u^{\mathrm{O}}_{not}$. It follows that u^{O}_{nos} are nondecreasing for $s < t^*$ in 1° of the first iteration. Furthermore, they keep nondecreasing even after 2°(b). Since $u^{\mathrm{O}}_{no,t-1} \geq y^{\mathrm{O}}_{not}$ from (33), 2° is terminated in a finite number of iterations with $y^{\mathrm{O}}_{not^*} = 0$ satisfied. When 2° is terminated, $u^{\mathrm{O}}_{no,t^*-1} \leq u^{\mathrm{O}}_{not^*}$ holds because $u^{\mathrm{O}}_{no,t-1} - y^{\mathrm{O}}_{not} \leq u^{\mathrm{O}}_{not}$ from (34). Thus, u^{O}_{nos} are nondecreasing for $s < t^*$ in 1° of the second and later iterations. In 1° of the final iteration, $u^{\mathrm{O}}_{not^*} = 0$ should be satisfied, so that (33) and (34) yield $y^{\mathrm{O}}_{not^*} = u^{\mathrm{O}}_{no,t^*-1}$. Therefore, $u^{\mathrm{O}}_{nos} = 0$ holds for any s when the procedure is terminated. It is not difficult to check that x^{O}_{nost} now satisfy (41) and (42). As we have already seen, (22)–(30) can be rewritten as (1)–(9), respectively, under (39)–(42).

□

5 NUMERICAL EXPERIMENT

In this section we examine the effectiveness of the proposed formulations (P1) and (P2) by numerical experiment. Test instances were generated randomly as follows. The number of units of product type p supplied by inbound truck m, a_{pm}, and the number of units of product type p requested from outbound truck n, b_{pn}, were generated from integer uniform distributions in $[1, 20]$, so that $\sum_{m=1}^{M} a_{pm} = \sum_{n=1}^{N} b_{pn}$ holds for every product type p. The release date of inbound truck m, r_m, and that of outbound truck n, l_n, were generated from integer uniform distributions in $[0, T-1]$. The duedate of outbound truck n, d_n, was generated from an integer uniform distribution in $[l_n+1, T]$. We assume $M = N$ and $I = J$, and M, I, P, T were chosen as $M \in \{5, 10\}$, $I \in \{2, 3\}$, $P \in \{2, 3\}$, and $T \in \{N, 2N\}$. For each combination of $M(=N)$, $I(=J)$, P, and T, five instances were generated. In all these instances, the travel time from inbound door i to outbound door o was set to $t_{io} = |i-o|$. We solved instances using (P1) and (P2) with capacity C changed as $C = 20$, 30, 40 and ∞. Gurobi Optimizer v7.5.2 [11] was used as an ILP solver. The computation was conducted on a desktop computer with an Intel core i7-6700 CPU (3.4GHz) and 24GB RAM. The time limit was set to 30 minutes for each instance.

The results are summarized in Tables 1 and 2, where the column "opt" denotes the number of instances solved to optimality within the time limit, and "time" denotes the average computation time in seconds over instances solved to optimality. We can observe from the tables that the small-size instances with $M = N = 5$ are easy to solve to optimality regardless of capacity C. In the case of $M = N = 10$, most instances can be solved to optimality, but not all. We can also see that instances with a smaller C are harder to solve to optimality than those with a larger C. In particular, all the instances without capacity constraints ($C = \infty$) were solved to optimality by both (P1) and (P2). With regard to the computation time of (P1) and (P2), (P2) yielded better results than (P1) in most cases. Indeed, as presented in Table 3, the number of instances with $M = N = 10$ solved to optimality by (P2) is $128 + 14 = 142$, whereas that by (P1) is $128 + 1 = 129$.

6 CONCLUSION

In this study we proposed two types of ILP formulation for the truck-to-door scheduling problem in a multi-door cross-docking terminal where temporary storage is taken into account. The first formulation employs binary decision variables indexed by arrival and departure times of trucks. The second formulation derives from those of the unit commitment problem and uses binary decision variables representing states of trucks as well as departure times. Numerical experiment showed that the latter formulation yields shorter computation time than the former, primarily due to fewer binary decision variables. Nevertheless, the latter formulation failed in solving some instances with 10 inbound trucks and 10 outbound trucks. To solve larger-size instances to optimality as well as these, it is necessary to construct dedicated exact algorithms based on branch-and-bound, branch-and-cut, and so on. It will also be worthwhile to construct heuristics such as local search algorithms and Lagrangian heuristics. These topics are left for future research.

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TABLE 1. Numerical results by (P1) and (P2) ($M = N = 5$)

I $(=J)$	P	T	C	(P1) opt	(P1) time (s)	(P2) opt	(P2) time (s)
2	2	5	20	5	0.03	5	0.03
			30	5	0.09	5	0.08
			40	5	0.05	5	0.04
			∞	5	0.04	5	0.04
		10	20	5	0.92	5	0.69
			30	5	0.37	5	0.27
			40	5	0.39	5	0.17
			∞	5	0.22	5	0.14
	3	5	20	5	0.19	5	0.19
			30	5	0.30	5	0.24
			40	5	2.08	5	1.09
			∞	5	0.06	5	0.04
		10	20	5	3.26	5	2.67
			30	5	12.63	5	9.39
			40	5	1.03	5	1.16
			∞	5	0.74	5	0.23
3	2	5	20	5	0.86	5	0.56
			30	5	1.78	5	1.99
			40	5	0.38	5	0.67
			∞	5	0.09	5	0.24
		10	20	5	1.46	5	1.59
			30	5	11.35	5	1.40
			40	5	1.60	5	0.92
			∞	5	0.92	5	0.25
	3	5	20	5	0.76	5	0.14
			30	5	0.26	5	0.44
			40	5	0.35	5	0.78
			∞	5	0.36	5	0.16
		10	20	5	267.32	5	14.13
			30	5	8.20	5	5.41
			40	5	3.14	5	0.97
			∞	5	1.11	5	0.49

TABLE 2. Numerical results by (P1) and (P2) ($M = N = 10$)

I $(=J)$	P	T	C	(P1) opt	(P1) time (s)	(P2) opt	(P2) time (s)
2	2	10	20	5	35.63	5	13.12
			30	5	83.48	5	23.91
			40	5	5.46	5	10.24
			∞	5	0.85	5	0.55
		20	20	*4*	*150.10*	*4*	*38.49*
			30	5	104.60	*4*	*6.99*
			40	5	2.98	5	1.19
			∞	5	0.74	5	0.39
	3	10	20	5	72.19	5	8.91
			30	*4*	*338.25*	5	427.16
			40	*4*	*123.53*	*4*	*190.73*
			∞	5	2.41	5	1.33
		20	20	*1*	*188.79*	*4*	*692.42*
			30	*2*	*350.26*	*4*	*548.97*
			40	*2*	*59.16*	*4*	*89.72*
			∞	5	13.67	5	4.88
3	2	10	20	*3*	*495.85*	*3*	*150.75*
			30	*4*	*483.23*	*4*	*127.54*
			40	*4*	*274.35*	*4*	*38.51*
			∞	5	2.83	5	1.05
		20	20	*3*	*495.85*	*3*	*150.75*
			30	*4*	*483.23*	*4*	*127.54*
			40	*4*	*274.35*	*4*	*38.51*
			∞	5	2.83	5	1.05
	3	10	20	*4*	*19.79*	*4*	*14.47*
			30	*3*	*937.40*	*4*	*249.77*
			40	*3*	*136.09*	*3*	*374.48*
			∞	5	9.42	5	4.00
		20	20	*2*	*872.26*	*2*	*189.06*
			30	*2*	*111.40*	*4*	*443.93*
			40	*4*	*325.75*	5	325.31
			∞	5	13.98	5	4.96

TABLE 3. The numbers of instances solved to optimality by (P1) and (P2)

trucks	both	only (P1)	only (P2)
$M = N = 5$	160	0	0
$M = N = 10$	128	1	14

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