Modeling and numerical simulations of fish like swimming

Michel Bergmann, Angelo Iollo

INRIA Bordeaux Sud-Ouest, équipe MC2

Institut de Mathématiques Appliquées de Bordeaux

33405 TALENCE cedex, France

Context and objectives

- **Context** : ANR CARPEiNETER *Cartesian grids, penalization and level set for the simulation and optimisation of complex flows*

- **Objectives**:

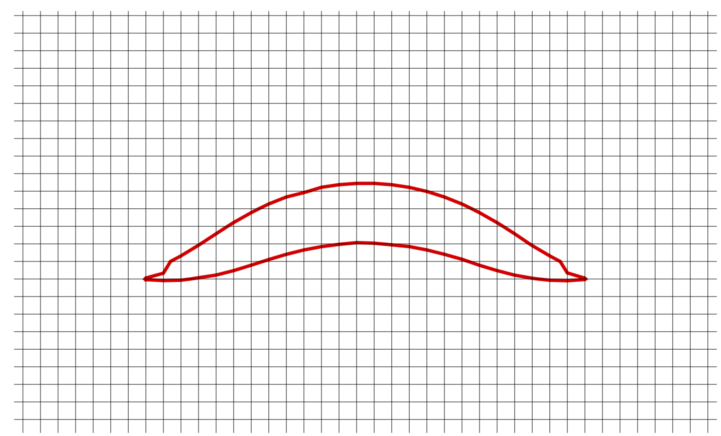
  ↪ Model and simulate moving bodies S (translation, rotation, deformation, ..)

  ↪ Couple Fluid and Structures

  ↪ **Cartesian meshes**
  Avoid remeshing

  ↪ **Penalization of the equations**
  To take into account the bodies

  ↪ **Level Set**
  To track interfaces
  (fluid/fluid, fluid/structures)



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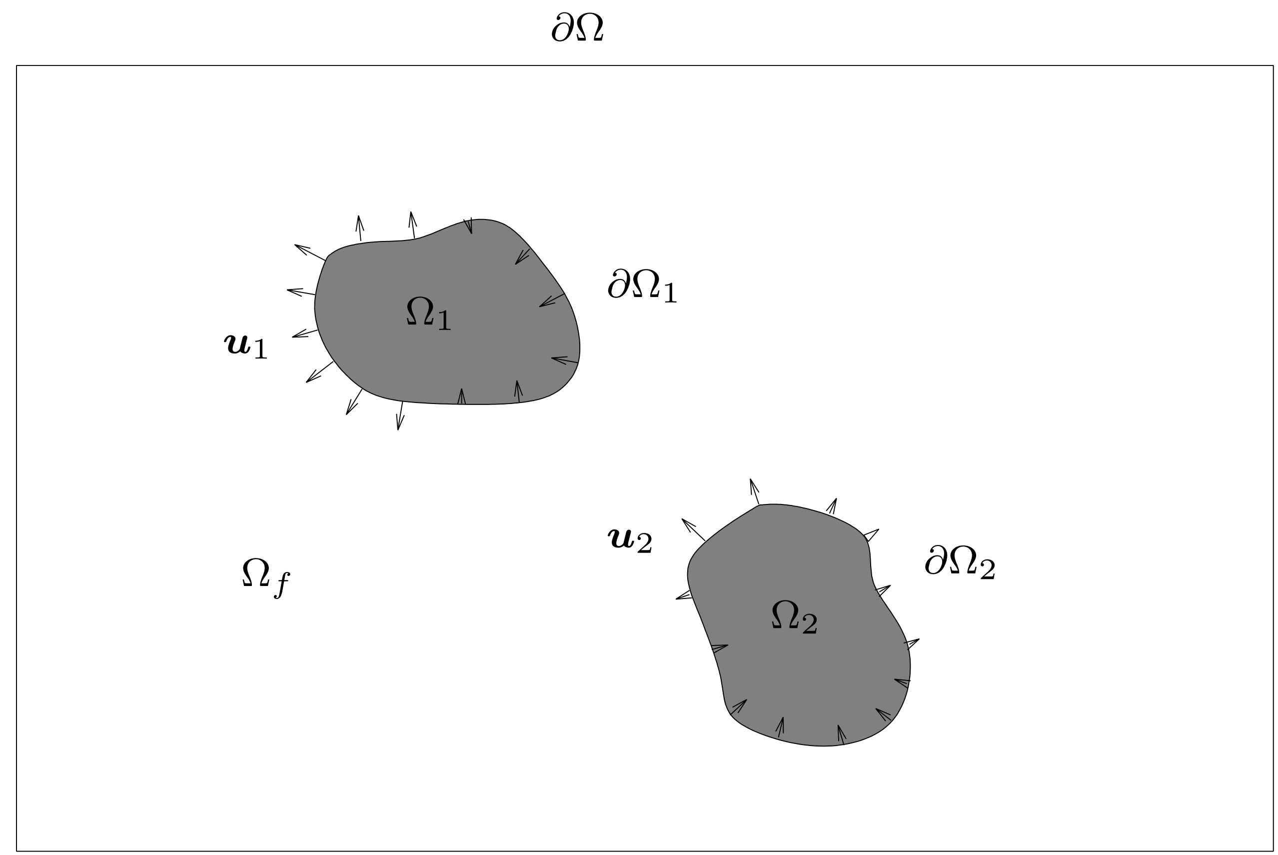
Outline

- **Flow modeling**
- **Numerical approach**
  - Method: discretization / body motion
  - Validation
- **Applications: 2D fish swimming**
  - Parametrization
  - Classification: BCF
  - On the power spent to swim
  - Maneuvers and turns
  - Fish school (3 fishes)
- **3D locomotion**
- **Conclusions and future works**

Flow modeling



Ω_i : Domain "body" i

Ω_f : Domain "fluid"

$\Omega = \Omega \cup \Omega_i$: Entire domain

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Flow modeling

- **Classical model:** Navier-Stokes equations (incompressible):

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right) = -\nabla p + \mu\Delta\boldsymbol{u} + \rho\boldsymbol{g} \quad \text{dans} \quad \Omega_f, \tag{1a}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \quad \text{dans} \quad \Omega_f, \tag{1b}$$

$$\boldsymbol{u} = \boldsymbol{u}_i \quad \text{sur} \quad \partial\Omega_i \tag{1c}$$

$$\boldsymbol{u} = \boldsymbol{u}_f \quad \text{sur} \quad \partial\Omega \tag{1d}$$

Numerical resolution

Need of meshes that fit the body geometries

↪ Costly remeshing for moving and deformable bodies!!

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Flow modeling

- **Penalization model:** penalized Navier-Stokes equations (incompressible):

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right) = -\nabla p + \mu\Delta\boldsymbol{u} + \rho\boldsymbol{g} + \lambda\rho\sum_{1=1}^{N_s}\chi_i(\boldsymbol{u}_i - \boldsymbol{u}) \quad \text{dans} \quad \Omega, \tag{2a}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \quad \text{dans} \quad \Omega, \tag{2b}$$

$$\boldsymbol{u} = \boldsymbol{u}_f \quad \text{sur} \quad \partial\Omega. \tag{2c}$$

$\lambda \gg 1$ penalization factor $\rightarrow$ Solution eqs (2) tends to solution eqs (1) *w.r.t.* $\varepsilon = 1/\lambda \rightarrow 0$.
χ_i characteristic function:

$$\chi_i(\boldsymbol{x}) = 1 \quad \text{if} \quad \boldsymbol{x} \in \Omega_i, \tag{3a}$$

$$\chi_i(\boldsymbol{x}) = 0 \quad \text{else if.} \tag{3b}$$

Numerical resolution

No need of meshes that fit the body geometries

$\hookrightarrow$ Cartesian meshes

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Flow modeling

- **Transport of the characteristic function** for moving bodies

$$\frac{\partial \chi_i}{\partial t} + (\boldsymbol{u}_i \cdot \nabla)\chi_i = 0. \tag{4}$$

Other choice: $\chi_i = H(\phi_i)$ where H is Heaviside function and ϕ_i the signed distance function ($\phi_i(\boldsymbol{x}) > 0$ if $\boldsymbol{x} \in \Omega_i$, $\phi_i(\boldsymbol{x}) = 0$ si $\boldsymbol{x} \in \partial\Omega_i$, $\phi_i(\boldsymbol{x}) < 0$ else if).

$$\frac{\partial \phi_i}{\partial t} + (\boldsymbol{u}_i \cdot \nabla)\phi_i = 0. \tag{5}$$

- **Density**

$$\widetilde{\rho} = \rho_f \left(1 - \sum_{i=1}^{N_s} \chi_i\right) + \sum_{i=1}^{N_s} \rho_i \chi_i. \tag{6}$$

Flow modeling

▶ **Dimensionless equations** with U_∞, D, ρ_f, $Re = \frac{\rho U_\infty D}{\mu}$:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\Delta \boldsymbol{u} + \boldsymbol{g} + \lambda \sum_{1=1}^{N_s} \chi_i (\boldsymbol{u}_i - \boldsymbol{u}) \quad \text{dans} \quad \Omega, \tag{7a}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \quad \text{dans} \quad \Omega, \tag{7b}$$

$$\boldsymbol{u} = \boldsymbol{u}_f \quad \text{sur} \quad \partial\Omega \tag{7c}$$

▶ **Body velocity $\boldsymbol{u}_i$:**

$$\boldsymbol{u}_i = \overline{\boldsymbol{u}}_i + \widehat{\boldsymbol{u}}_i + \widetilde{\boldsymbol{u}}_i \tag{8}$$

with:

$\overline{\boldsymbol{u}}_i$ translation velocity

$\widehat{\boldsymbol{u}}_i$ rotation velocity

$\widetilde{\boldsymbol{u}}$ deformation velocity (imposed for the swim)

Numerical approach | Method

- **Space:** Cartesian meshes, collocation with compact "non oscillating" scheme,
  Centered FD 2nd order and upwind 3rd order for convective terms

- **Time:** 1^{st} order explicit euler, implicit penalization (larger λ)

$$\frac{\boldsymbol{u}^{(n+1)} - \boldsymbol{u}^{(n)}}{\Delta t} + (\boldsymbol{u}^{(n)} \cdot \nabla)\boldsymbol{u}^{(n)} = -\nabla p^{(n+1)} + \frac{1}{Re}\Delta \boldsymbol{u}^{(n+1)} + \boldsymbol{g}$$
$$+\lambda \sum_{1=1}^{N_s} \chi_i{}^{(n+1)}(\boldsymbol{u}_i{}^{(n+1)} - \boldsymbol{u}^{(n+1)}),$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{u}^{(n+1)} = 0$$

$\Rightarrow$ **Problems**

$\hookrightarrow$ Pressure uncoupled

$\hookrightarrow$ The function $\chi_i{}^{(n+1)}$ and velocity $\boldsymbol{u}_i{}^{(n+1)}$ are not known

$\Rightarrow$ **Solutions**

$\hookrightarrow$ Chorin scheme (predictor/corrector)

$\hookrightarrow$ Fractional step method (2 steps)

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Numerical approach | Method

- **Fractional steps method**

$$\frac{\boldsymbol{u}^{(n+1)} - \boldsymbol{u}^{(n)}}{\Delta t} + (\boldsymbol{u}^{(n)} \cdot \nabla)\boldsymbol{u}^{(n)} = -\nabla p^{(*)} + \frac{1}{Re}\Delta \boldsymbol{u}^{(n+1)} + \boldsymbol{g}$$
$$+ \left(\nabla p^{(*)} - \nabla p^{(n+1)}\right)$$
$$+\lambda \sum_{1=1}^{N_s} \chi_i{}^{(n+1)}(\boldsymbol{u}_i{}^{(n+1)} - \boldsymbol{u}^{(n+1)}),$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u}^{(n+1)} = 0$$

$$\boldsymbol{u}_i^{(n+1)} = f(\boldsymbol{u}^{(n+1)}, p^{(n+1)})$$

Step 1: $\Rightarrow \boldsymbol{u}^{(*)}, p^{(*)}$

Step 2 : $\Rightarrow \widetilde{\boldsymbol{u}}^{(n+1)}, \widetilde{p}^{(n+1)}$

Step 3 : $\Rightarrow \boldsymbol{u}_i^{(n+1)} = \widetilde{f}(\widetilde{\boldsymbol{u}}^{(n+1)}, \widetilde{p}^{(n+1)})$

Step 4 : $\Rightarrow \boldsymbol{u}^{(n+1)}, p^{(n+1)}$

Numerical approach | Method

- **Step 1 :** prediction

$$\frac{\boldsymbol{u}^{(*)} - \boldsymbol{u}^{(n)}}{\Delta t} + (\boldsymbol{u}^{(n)} \cdot \nabla)\boldsymbol{u}^{(n)} = -\nabla p^{(*)} + \frac{1}{Re}\Delta \boldsymbol{u}^{(*)} + \boldsymbol{g}$$

- **Step 2 :** correction

$$\frac{\widetilde{\boldsymbol{u}}^{(n+1)} - \boldsymbol{u}^{(*)}}{\Delta t} = \nabla p^{(*)} - \nabla p^{(n+1)}$$

$$\boldsymbol{\nabla} \cdot \widetilde{\boldsymbol{u}}^{(n+1)} = 0$$

with $\psi = \nabla p^{(*)} - \nabla p^{(n+1)}$, on a $\Delta \psi = \nabla \cdot \boldsymbol{u}^{(*)}$

$$\widetilde{\boldsymbol{u}}^{n+1} = \widetilde{\boldsymbol{u}}^{*} - \nabla \psi$$

$$\widetilde{p}^{n+1} = \widetilde{p}^{*} + \frac{\psi}{\Delta t}$$

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Numerical approach | Method

▶ **Etape 3 :** body motion *Compute forces* $\boldsymbol{F}_i$ *and torques* $\boldsymbol{\mathcal{M}}_i$

$$m\frac{\mathrm{d}\overline{\boldsymbol{u}}_i}{\mathrm{d}t} = \boldsymbol{F}_i + m\boldsymbol{g}, \qquad \overline{\boldsymbol{u}}_i \text{ translation velocity}, \quad m \text{ mass} \tag{14a}$$

$$\frac{\mathrm{d}J\boldsymbol{\Omega}_i}{\mathrm{d}t} = \boldsymbol{\mathcal{M}}_i, \qquad \boldsymbol{\Omega}_i \text{ angular velocity}, \quad J \text{ inertia matrix} \tag{14b}$$

Rotation velocity $\widehat{\boldsymbol{u}}_i = \boldsymbol{\Omega}_i \times \boldsymbol{r}_G$ with $\boldsymbol{r}_G = \boldsymbol{x} - \boldsymbol{x}_G$ ($\boldsymbol{x}_G$ center of mass).
Stress tensor $\mathbb{T}(\boldsymbol{u}, p) = -p\mathbb{I} + \frac{1}{Re}(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)$ et n outward normal unit vector at s_i:

$$\boldsymbol{F}_i = -\int_{\partial\Omega_i} \mathbb{T}(\boldsymbol{u}, p)\,\boldsymbol{n}\,\mathrm{d}\boldsymbol{x}, \tag{15a}$$

$$\boldsymbol{\mathcal{M}}_i = -\int_{\partial\Omega_i} \mathbb{T}(\boldsymbol{u}, p)\,\boldsymbol{n} \times \boldsymbol{r}_G\,\mathrm{d}\boldsymbol{x}. \tag{15b}$$

Evaluation of forces and torques

Cartesian mesh: no direct acces to $\partial\Omega_i$

$\hookrightarrow$ Not easy evaluation ....

Numerical approach | Method

Definition : Arbitrarily domain $\Omega_{f_i}(t)$ surrounding body i.

Forces:

$$\boldsymbol{F}_i = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega_{f_i}(t)} \boldsymbol{u}\,\mathrm{d}V + \int_{\partial\Omega_{f_i}(t)} (\mathbb{T} + (\boldsymbol{u}-\boldsymbol{u}_i)\otimes\boldsymbol{u})\,\boldsymbol{n}\,\mathrm{d}S + \int_{\partial\Omega_i(t)} ((\boldsymbol{u}-\boldsymbol{u}_i)\otimes\boldsymbol{u})\,\boldsymbol{n}\,\mathrm{d}S. \tag{16a}$$

Torques:

$$\boldsymbol{\mathcal{M}}_i = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega_{f_i}(t)} \boldsymbol{u}\times\boldsymbol{r}_G\,\mathrm{d}V + \int_{\partial\Omega_{f_i}(t)} (\mathbb{T} + (\boldsymbol{u}-\boldsymbol{u}_i)\otimes\boldsymbol{u})\,\boldsymbol{n}\times\boldsymbol{r}_G\,\mathrm{d}S + \int_{\partial\Omega_i(t)} ((\boldsymbol{u}-\boldsymbol{u}_i)\otimes\boldsymbol{u})\,\boldsymbol{n}\times\boldsymbol{r}_G\,\mathrm{d}S. \tag{16b}$$

Evaluation of forces and torques

The term onto $\partial\Omega_i$ vanishes in our case (no transpiration)

$\hookrightarrow$ Easy evaluation!

Numerical approach | Method

▶ **Step 4 :** Update velocity using implicit penalization

$$\frac{\boldsymbol{u}^{(n+1)} - \widetilde{\boldsymbol{u}}^{(n+1)}}{\Delta t} = \lambda \sum_{1=1}^{N_s} \chi_i{}^{(n+1)} (\boldsymbol{u}_i{}^{(n+1)} - \boldsymbol{u}^{(n+1)})$$

▶ **Summary:**

▷ Solve Navier-Stokes equation without penalization $\Rightarrow \widetilde{\boldsymbol{u}}^{(n+1)}$, $\widetilde{p}^{(n+1)}$

▷ Compute body motion $\Rightarrow \boldsymbol{u}_i^{(n+1)}$, $\chi_i^{(n+1)}$

▷ Correct solution with penalization $\Rightarrow \boldsymbol{u}^{(n+1)}$, $p^{(n+1)}$

▶ **Remark:**

▷ Step 4 can be implemented in step 1using explicit body velocity (time order is 1).

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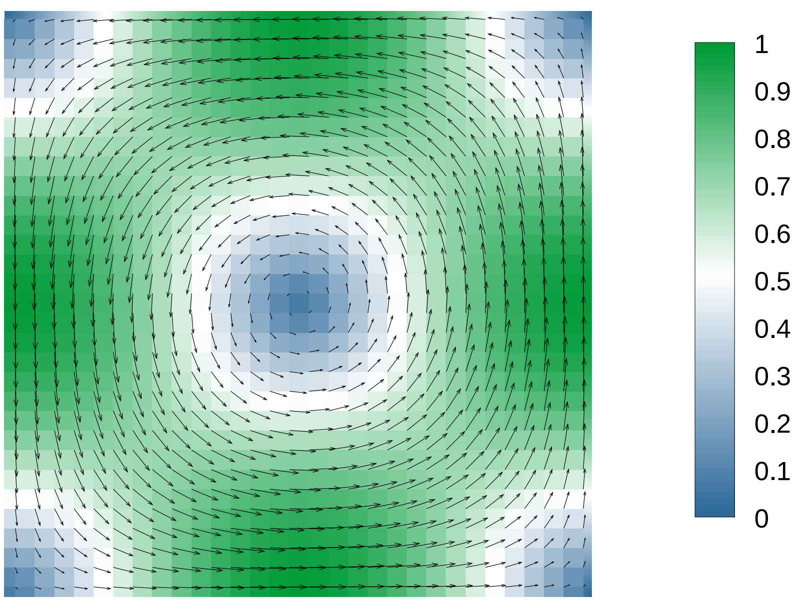
Numerical approach | Validation

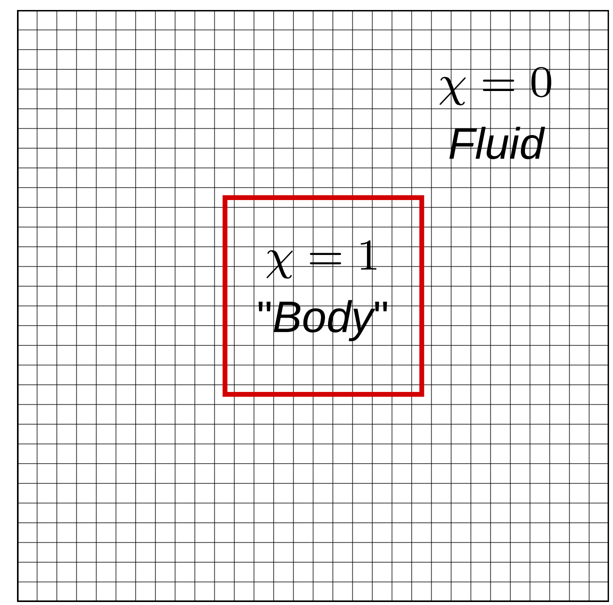
▶ **Improvement of the penalization order**

↪ Test case: 2D Green-Taylor vortex with analytical solution ($0 \leq x,\ y \leq \pi$, $Re = 100$)

$$\begin{aligned} u(t,\,\boldsymbol{x}) &= \sin(x)\cos(y)\exp(-2t/Re), \\ v(t,\,\boldsymbol{x}) &= -\cos(x)\sin(y)\exp(-2t/Re), \\ p(t,\,\boldsymbol{x}) &= \frac{1}{4}(\cos(2x)+\cos(2y))\exp(-4t/Re). \end{aligned}$$

$$E = \sqrt{\int_\Omega (\widetilde{\boldsymbol{u}}(T_f,\,\boldsymbol{x}) - \boldsymbol{u}(T_f,\,\boldsymbol{x}))^2\,\mathrm{d}x}.$$





↪ *"Non intrusive" body ⇒ penalization velocity depends on space and time*

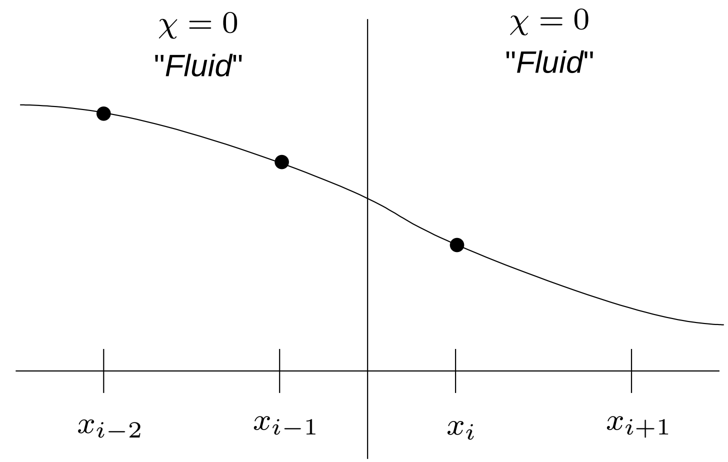
Numerical approach | Validation

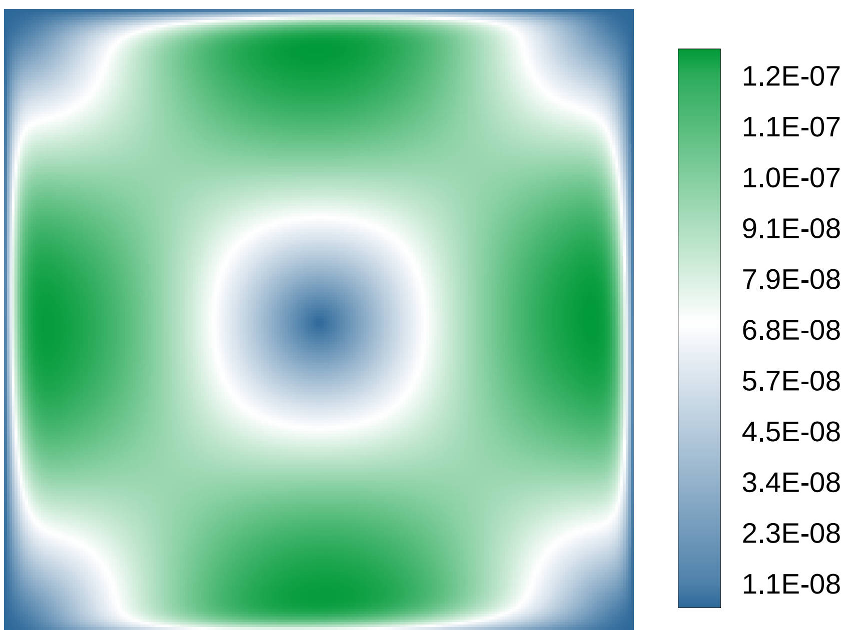
1 - No penalization

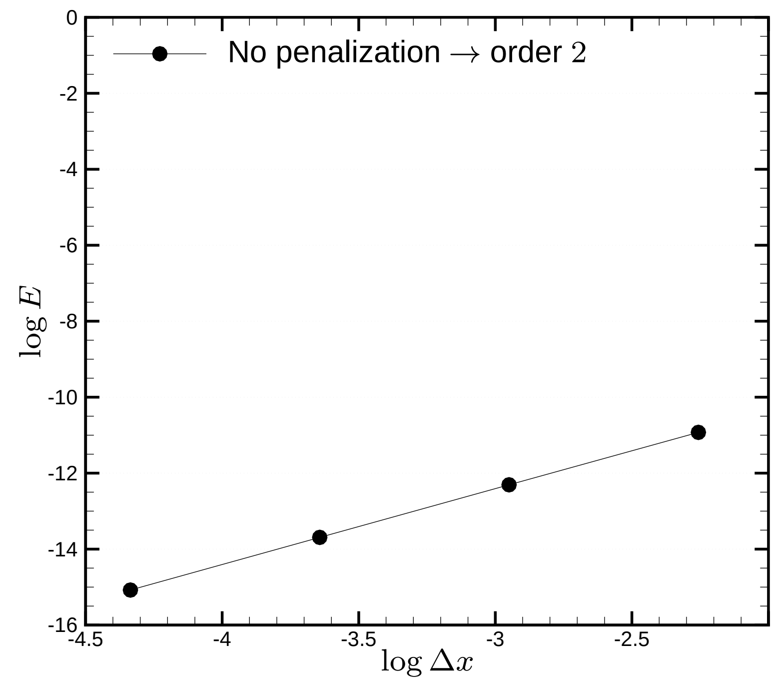
$\hookrightarrow$ use analytical boundary conditions

$\rightarrow$ Numerical scheme order, $(\Delta x)^2$

$\Rightarrow 2^{nd}$ *order*







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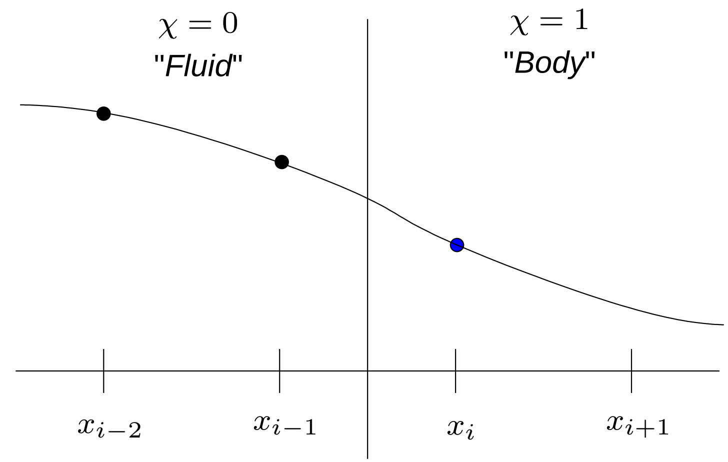
Numerical approach | Validation

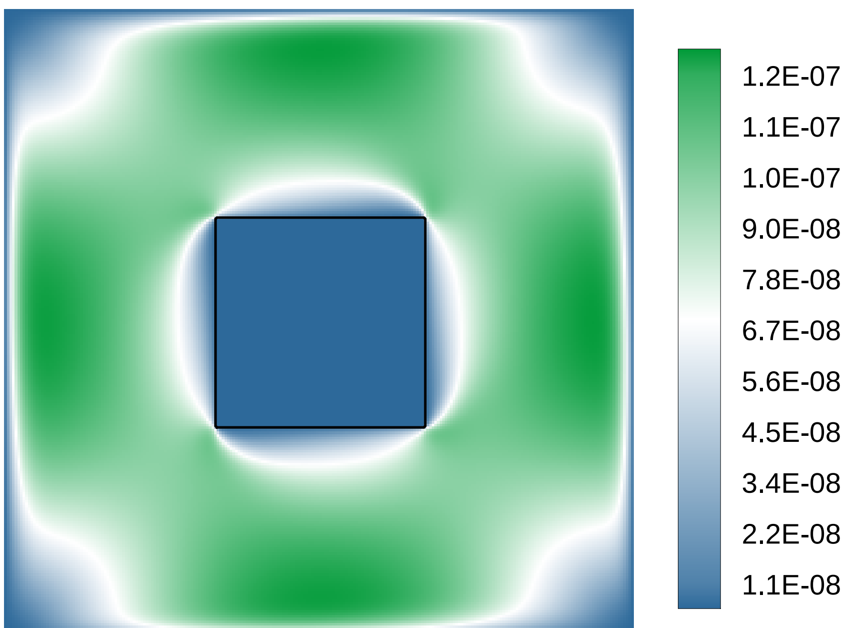
2 - "Exact" penalization:

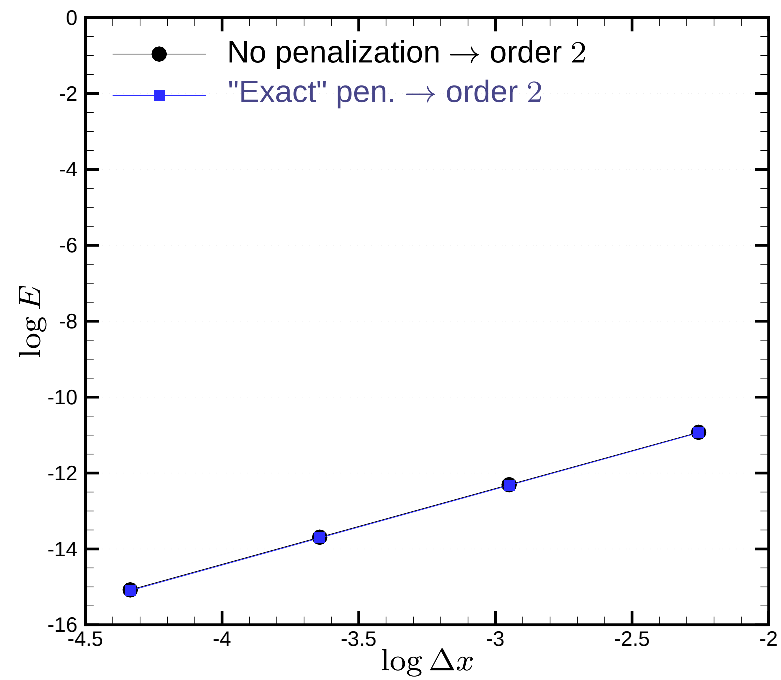
$\hookrightarrow$ use analytical penalization values

$$\overline{\boldsymbol{u}}_i^n = \boldsymbol{u}_i^n$$

$\Rightarrow 2^{nd}$ *order*







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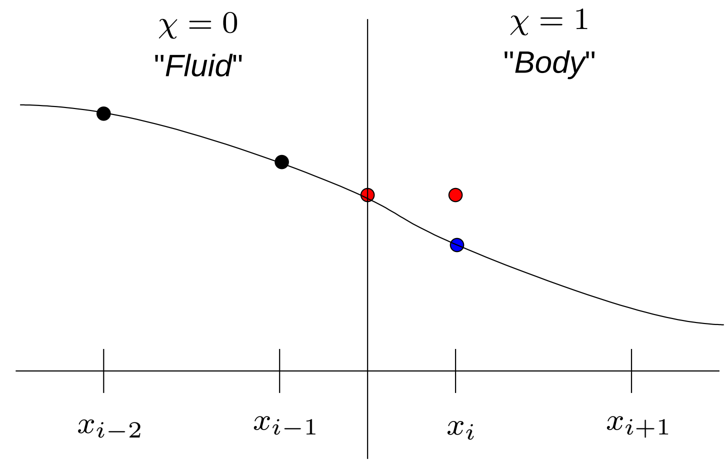
Numerical approach | Validation

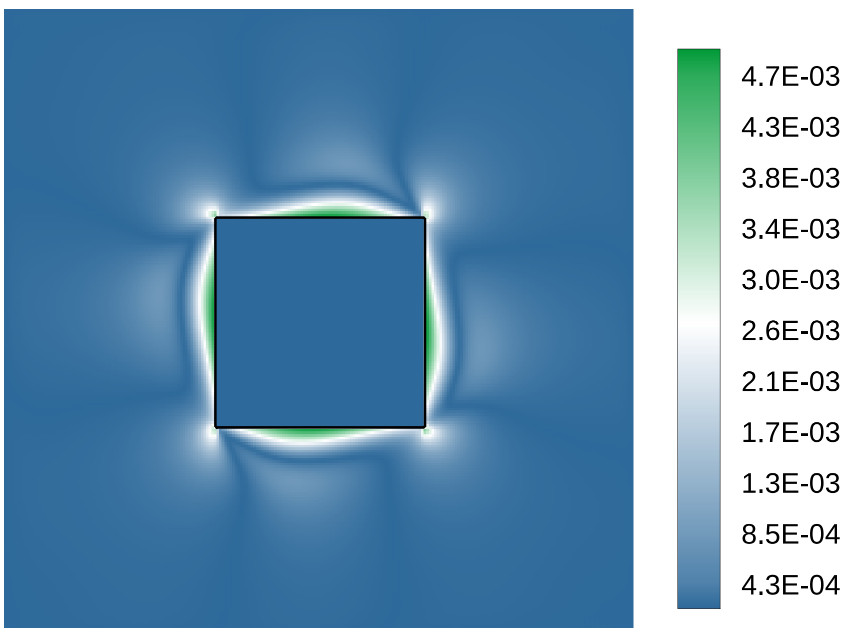
3 - "Standard" penalization:

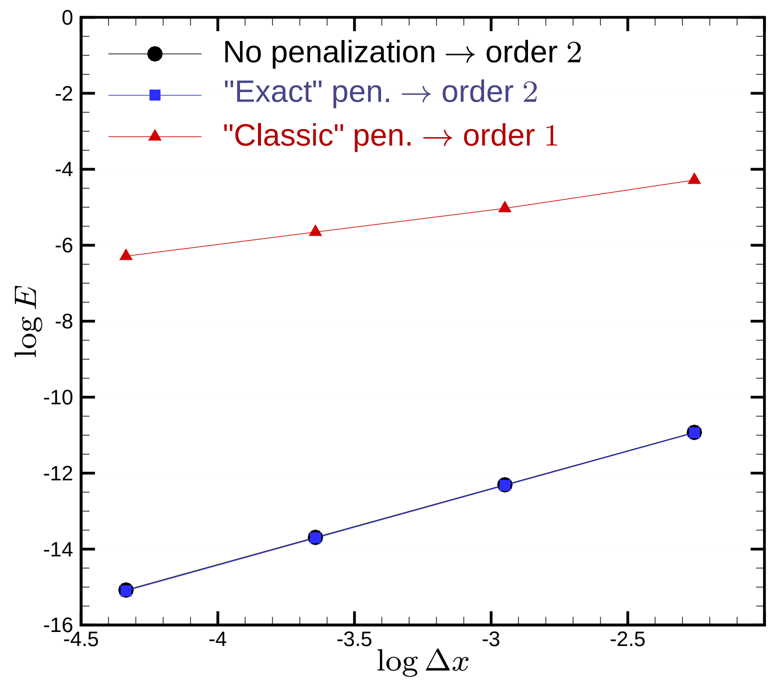
$\hookrightarrow$ use only boundary velocity

$$\overline{\boldsymbol{u}}_i^n = \boldsymbol{u}_{\phi=0}^n$$

$\Rightarrow 1^{nd}$ *order*







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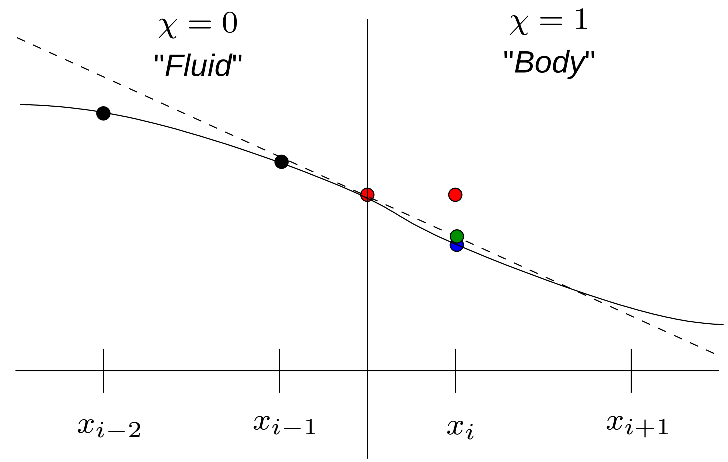
Numerical approach | Validation

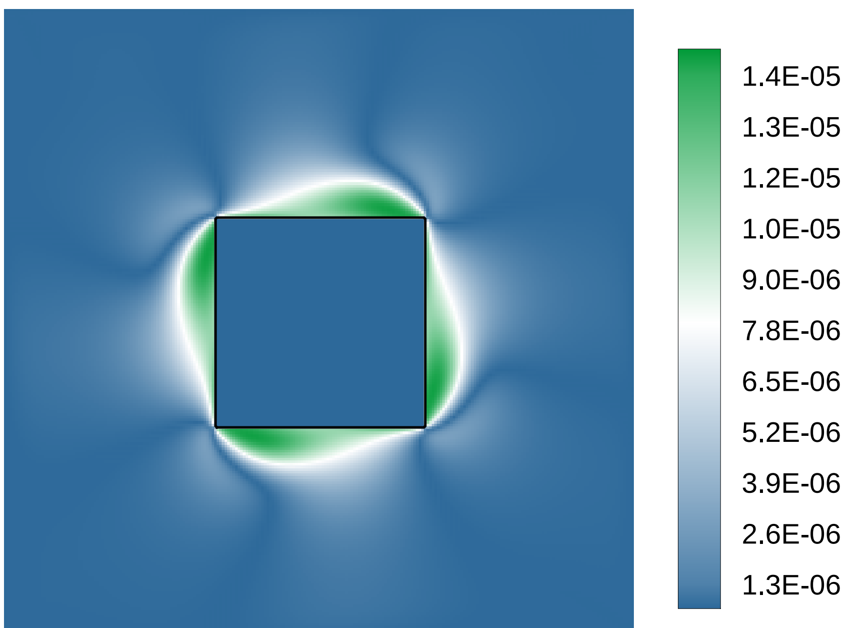
4 - "Improved" penalization:

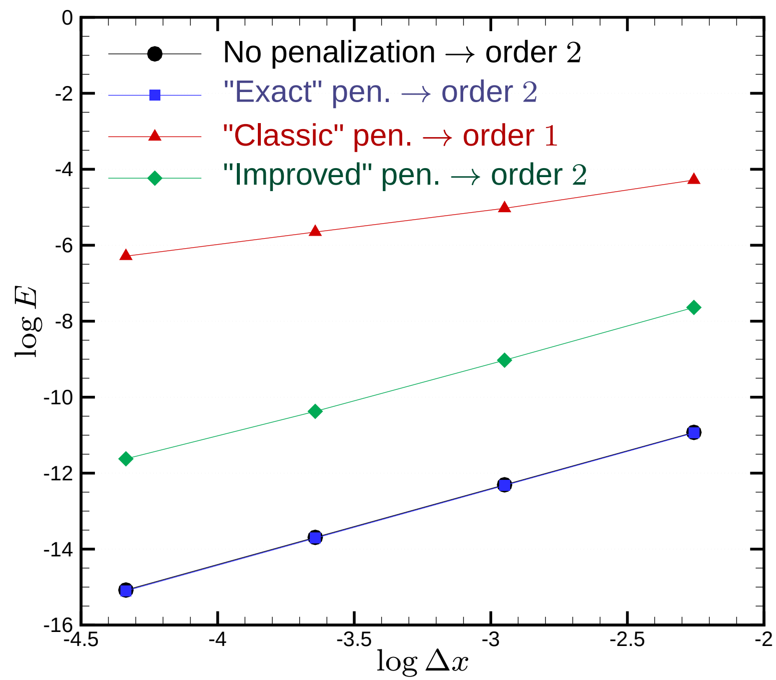
$\hookrightarrow$ use Level Set informations

$$\overline{\boldsymbol{u}}_i^n = \boldsymbol{u}_{\phi=0}^n - \phi_i \left(\partial \boldsymbol{u}_i / \partial \boldsymbol{n}\right)^{n-1}$$

$\Rightarrow 2^{nd}$ *order*







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Numerical approach | Validation

▸ **Validation 1: steady cylinder at $Re = 200$:**

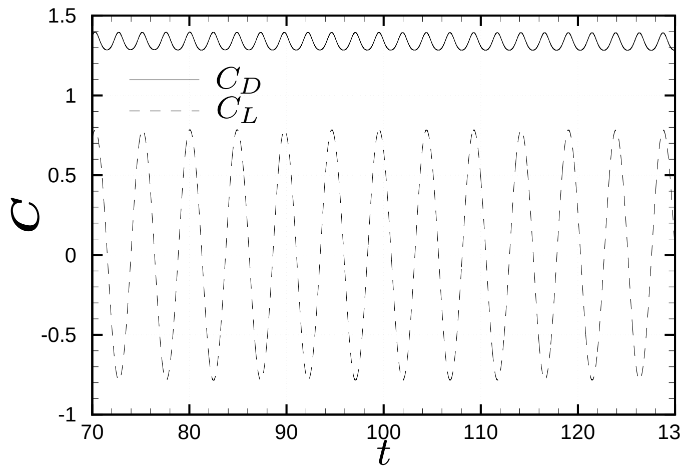

Fig. : *Temporal evolution of the lift (dashed line) and the drag (solid line) at* $Re = 200$.

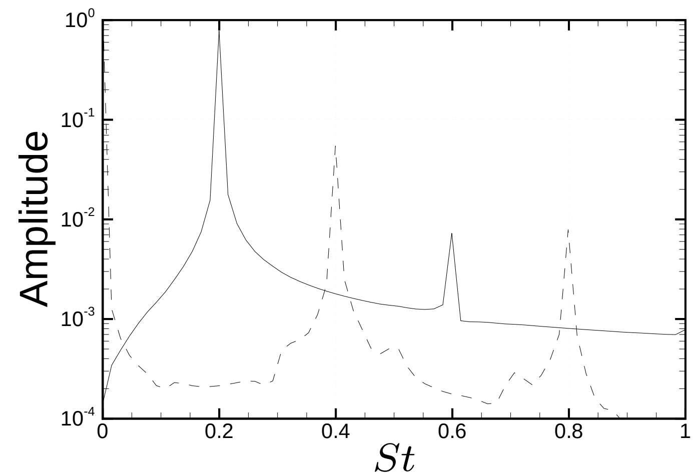

Fig. : *Spectrum (DFT) of the lift (dashed line) and the drag (solid line) at* $Re = 200$.

Authors	S_t	C_D
Braza 1986	0,2000	1,4000
Henderson 1997	0,1971	1,3412
He *et al.* 2000	0,1978	1,3560
Bergmann 2006	0,1999	1,3900
Présente étude	0,1980	1,3500

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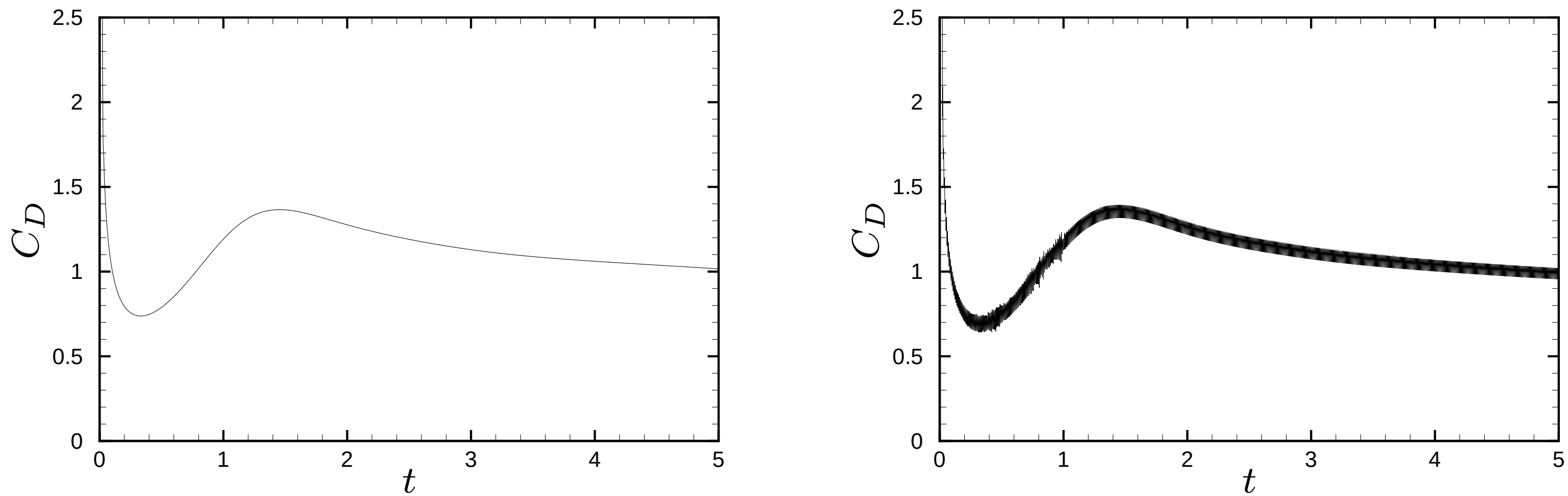
Numerical approach | Validation

- **Validation 2: moving cylinder at $Re = 550$:**

u_∞ is velocity at infinity, $\overline{u}_s$ is cylinder velocity



(a) $u_\infty = 1$, $\overline{u}_s = 0$. (b) $u_\infty = 0$, $\overline{u}_s = -1$.

Fig. : *Drag coefficient for an impulsively started cylinder at $Re = 550$. Medium time.*

$\hookrightarrow$ Similar results that those obtained by `Ploumhans` *`et al.`* `JCP` **`165`** `(2010)`

Remark: The oscillations (b) decrease with order and mesh refinement, `Chiu` *`et al.`* `JCP` **`229`** `2010`

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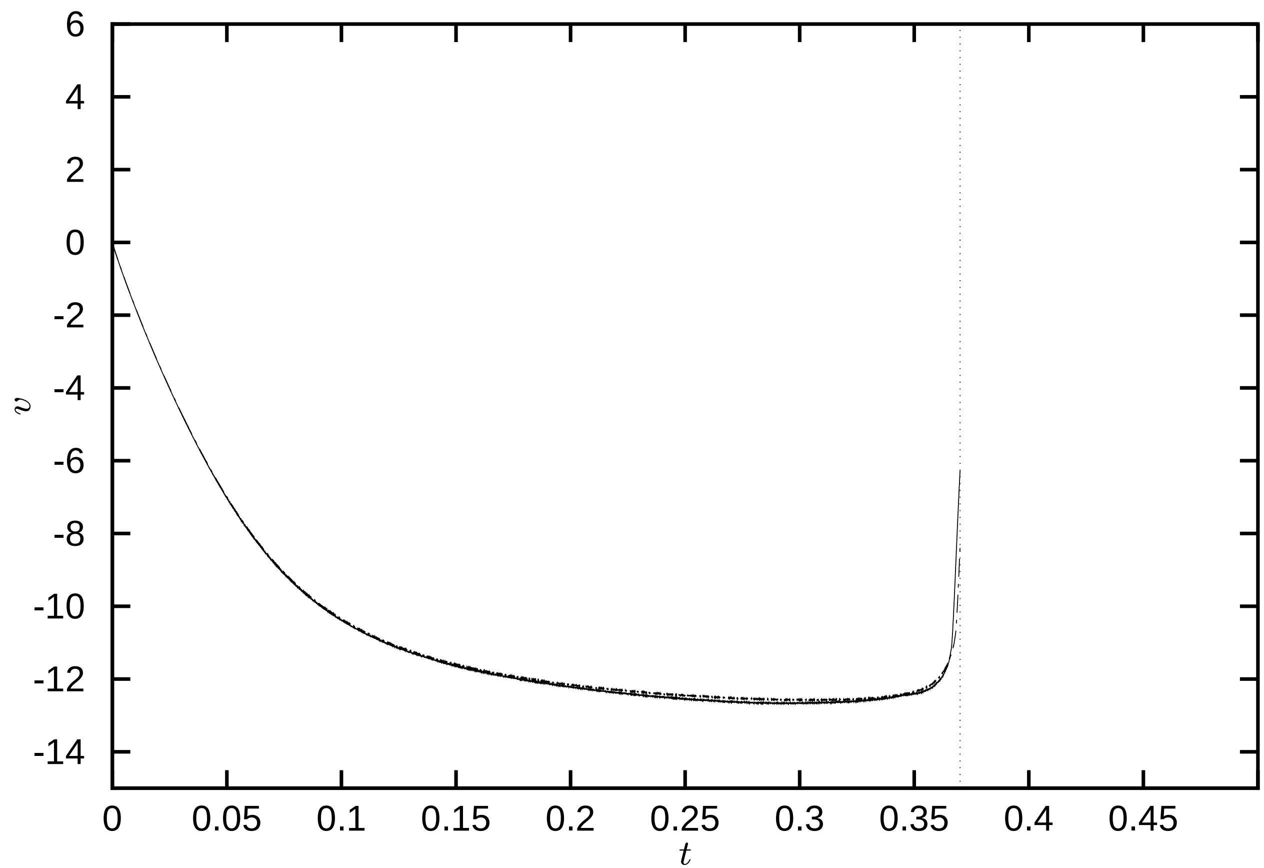
Numerical approch | Validation

- **Validation 3: Sedimentation of a cylinder (2D + gravity + rigid):**



$\hookrightarrow$ Similar results Refs. $[1, 2] \Rightarrow$ Validation

[1] M. Coquerelle, G.-H. Cottet, JCP **227** (2008)

[2] R. Glowinski, *et al.*, JCP **169** (2001)

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Fish swimming | Parametrization

▶ **Body velocity i:**

$$\boldsymbol{u}_i = \overline{\boldsymbol{u}}_i + \widehat{\boldsymbol{u}}_i + \widetilde{\boldsymbol{u}}_i. \tag{18}$$

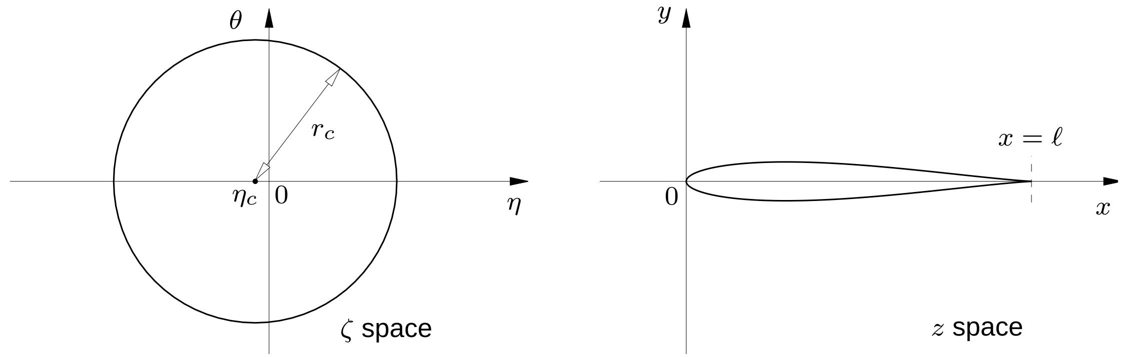
- Translation velocity $\overline{\boldsymbol{u}}_i$ is computed using forces $\boldsymbol{F}$
- Rotation velocity $\widehat{\boldsymbol{u}}_i$ is computed using torques $\boldsymbol{\mathcal{M}}$
- Deformation velocity $\widetilde{\boldsymbol{u}}_i$ is imposed for the swim

▷ Take care to not add artificial forces and torques!

1. Impose any deformation,
2. Subtract mass center deplacement,
3. Rotate the body by the opposite angle generate by deformation ,
4. Homothety for mass conservation

Fish swimming | Parametrization

- **Steady fish shape:**



(c) Original shape (d) Steady shape

Fig. : *Sketch of the Karman-Trefftz transform. The z space is transformed to fit* $0 \leq x_s \leq \ell$

$$z = n \frac{\left(1 + \frac{1}{\zeta}\right)^n + \left(1 - \frac{1}{\zeta}\right)^n}{\left(1 + \frac{1}{\zeta}\right)^n - \left(1 - \frac{1}{\zeta}\right)^n},$$

$\Rightarrow$ **Only 3 parameters** $\boldsymbol{b} = (\eta_c,\, \alpha,\, \ell)^T$

$\triangleright\ \alpha = (2 - n)\pi$: tail angle

$\triangleright\ \eta_c < 0$ circle center

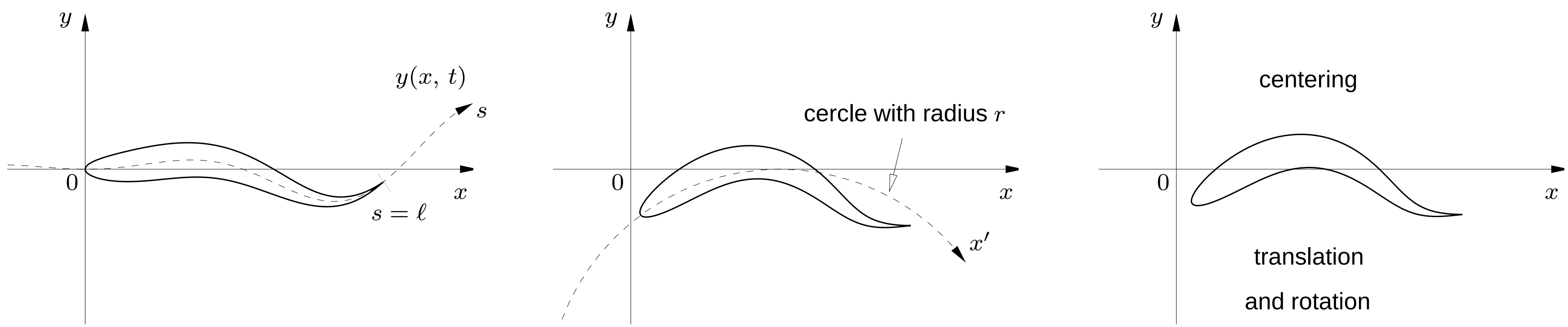
$\triangleright\ \ell > 0$ fish length ($\ell = 1$)

Fish swimming | **Parametrization**

▸ **Unsteady deformation of fish shape:** swimming law

↪ Backbone deformation: $s = \int_{x_0}^{x} \left(1 + \left(\frac{\partial y(x', t)}{\partial x'}\right)^2\right) \mathrm{d}x'$.

$$y(x,\, t) = (c_1 x + c_2 x^2) \sin(2\pi(x/\lambda + ft)). \qquad (19)$$



(e) Swimming shape (f) Maneuvering shape (g) Real motion shape

Fig. : *Sketch of swimming and maneuvering shape.*

⇒ **Only 4 parameters** $\boldsymbol{s} = (c_1,\, c_2,\, \lambda,\, f)^T$

▹ 2 parameters for envelop curve c_1 et c_2 + frequency f + wavelength λ.

⇒ **Shape** $\boldsymbol{b} = (\eta_c,\, \alpha,\, \ell)^T$ **+ swimming law** $\boldsymbol{s} = (c_1,\, c_2,\, \lambda,\, f)^T$ **= 7 parameters**

(we can also add $r(t)$ for maneuvers)

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Fish swimming| Wake organization

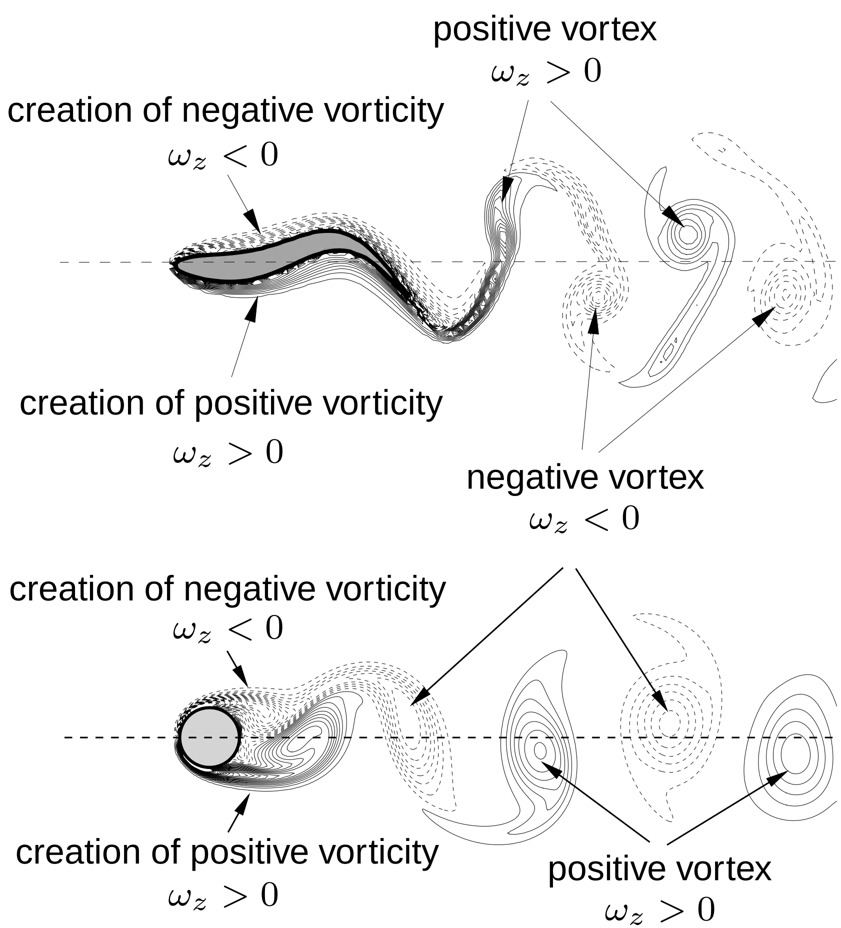

Fig. : *Inverted von Karman street.*

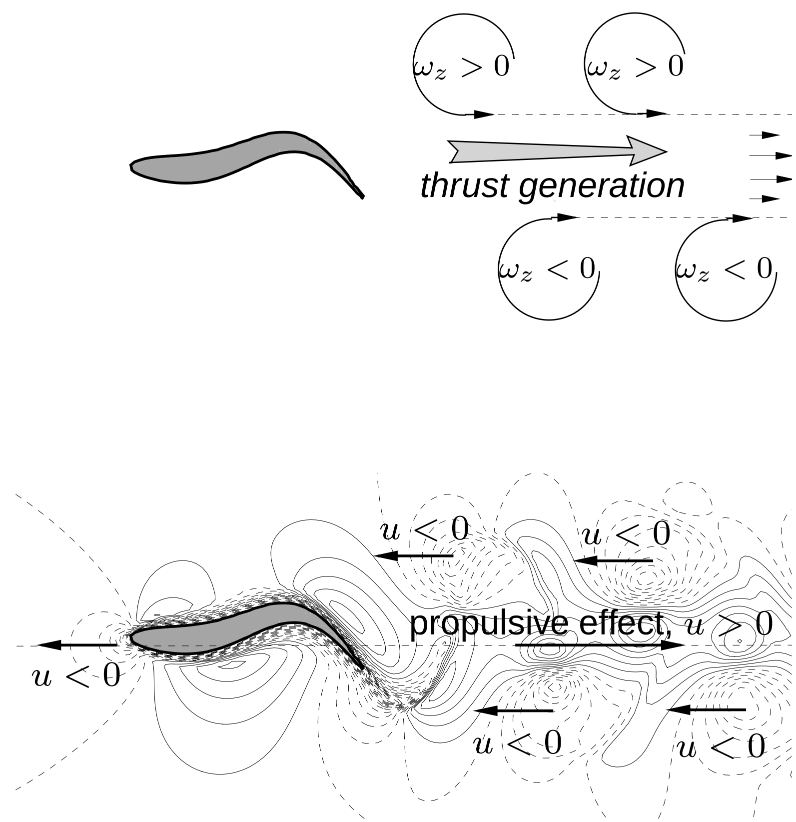

Fig. : *Propulsive effect.*

Fish swimming | Classification of fishes

- Fishes classified into 2 categories :
  - Median and Paired Fins (MPF)
  - Body and Caudal Fin (BCF) : most common
    - ↪ Thunniform (approx. par F_1)
    - ↪ Carangiform (approx. par F_2)
    - ↪ Subcarangiform (approx. par F_3)
    - ↪ Anguiliform (approx. par F_4)

Fish	Shape			swimming law			
Fi	η_c	α	ℓ	c_1	c_2	λ	f
F_1	-0.04	5	1	0.1	0.9	1.25	2
F_2	-0.03	5	1	0.4	0.6	1.00	2
F_3	-0.02	5	1	0.7	0.3	0.75	2
F_4	-0.01	5	1	1.0	0.0	0.50	2

Tab. : *Numerical parameters. The maximal tail amplitude deformation is* $A(c_1, c_2, \ell) = 0.4$.

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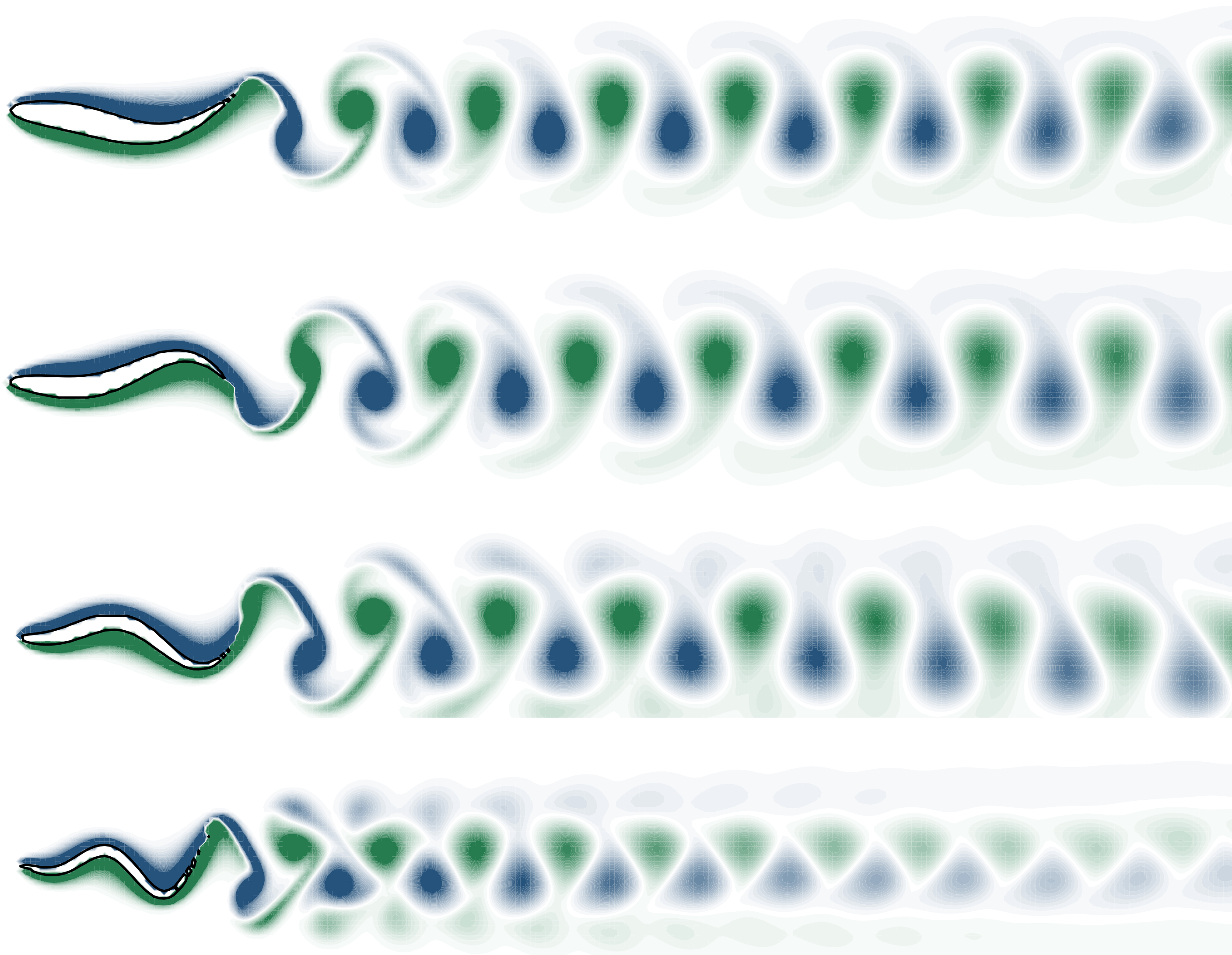
Fish swimming | BCF modes

Fish F_1

Fish F_2

Fish F_3

Fish F_4



Comparison of wakes generated at $Re = 10^3$

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Fish swimming | BCF modes

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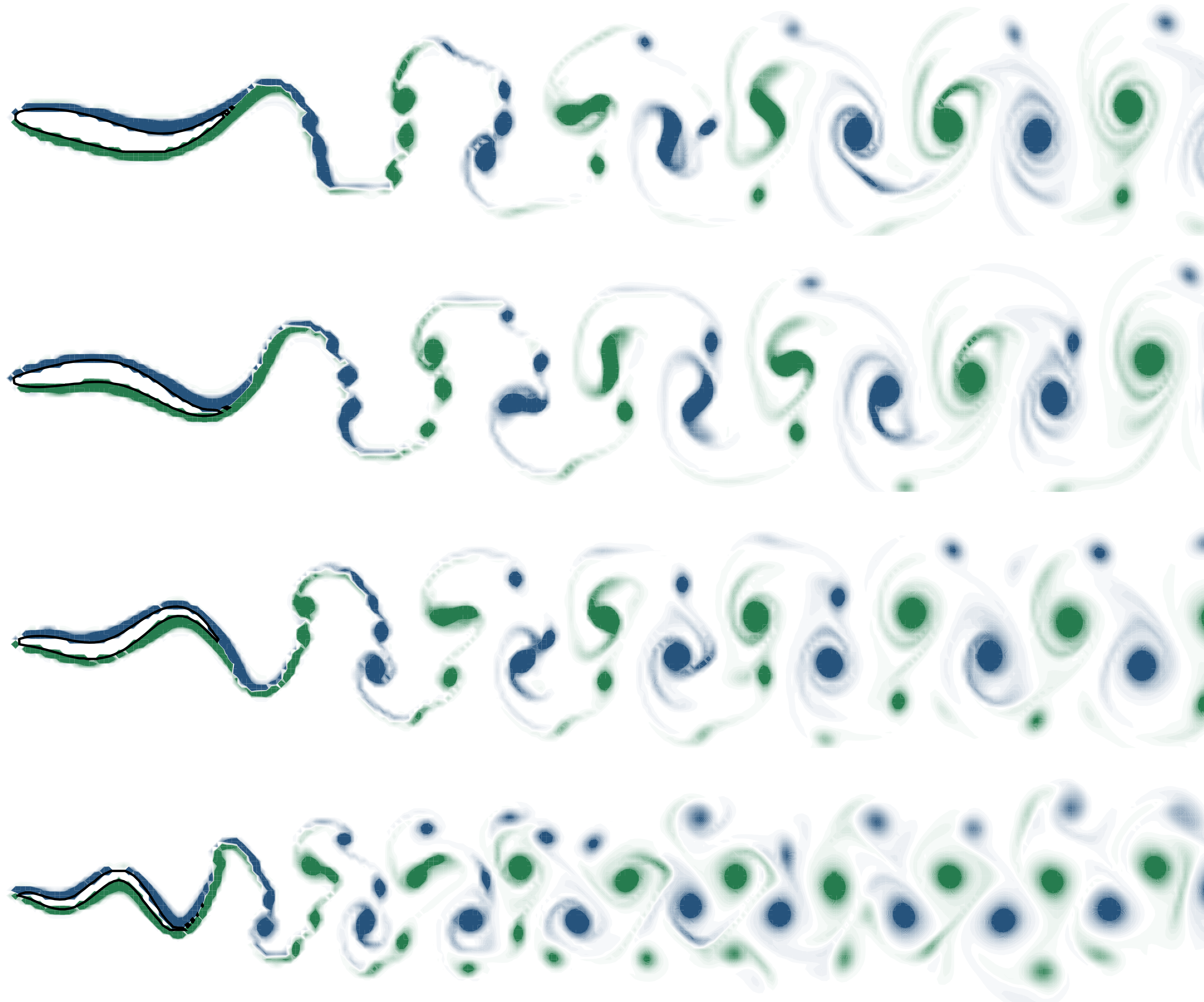
Fish swimming | BCF modes

Fish F_1

Fish F_2

Fish F_3

Fish F_4



Comparison of wakes generated at $Re = 10^4$

Fish swimming | BCF modes

Fish swimming | BCF modes

- Each fish swims on distance $D = 9$
  - $\hookrightarrow$ $|U_{max}|$: maximal velocity
  - $\hookrightarrow$ $|\overline{U}|$: mean velocity
  - $\hookrightarrow$ $|\gamma_{max}|$: maximal acceleration
  - $\hookrightarrow$ T_9: time to reach distance $D = 9$

	$Re = 10^3$				$Re = 10^4$			
$fish$	$\lvert U_{max}\rvert$	$\lvert\overline{U}\rvert$	$\lvert\gamma_{max}\rvert$	T_9	$\lvert U_{max}\rvert$	$\lvert\overline{U}\rvert$	$\lvert\gamma_{max}\rvert$	T_9
F_1	0.91	0.83	3.3	10.81	1.42	1.22	3.4	7.37
F_2	0.97	0.93	4.6	9.70	1.39	1.27	4.9	7.06
F_3	0.92	0.89	7.5	10.13	1.18	1.14	8.0	7.88
F_4	0.65	0.63	9.5	14.2	0.81	0.79	10.4	11.4

Tab. : *Maximal velocity* $|U_{max}|$, *maximal acceleration* $|\gamma_{max}|$ *and average velocity* $|\overline{U}|$ *at* $Re = 10^3$ *and* $Re = 10^4$.

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Fish swimming | Power spent

- The power spent to swim is:

$$P(t) = -\int_{\partial\Omega_s} p\,\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S + \int_{\partial\Omega_s} (\sigma'\cdot\boldsymbol{n})\cdot\boldsymbol{u}\,\mathrm{d}S, \tag{20}$$

with

$$\sigma'_{ij} = \frac{1}{Re}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)$$

- Transformation using energy conservation (remove $\partial\Omega_s$)

$$P(t) = \frac{\partial}{\partial t}\int_{\Omega_f} \frac{u^2}{2}\,\mathrm{d}\Omega + \frac{1}{Re}\int_{\Omega_f} \sigma'_{ij}\frac{\partial u_i}{\partial x_j}\,,\mathrm{d}\Omega. \tag{21}$$

$\hookrightarrow$ power = kinetic energy variation + power lost in viscous dissipation

Fish swimming | Power spent

- **Average energy:**

  $\hookrightarrow$ Energy for fish F_k to swim distance D is $E^{(k)} = \int_{T_k} P^{(k)}\,\mathrm{d}t$.

$Poisson$	$Re = 10^3$	$Re = 10^4$
F_1	0.98	0.60
F_2	0.99	0.54
F_3	0.90	0.45
F_4	0.77	0.30

Tab. : *Comparison of the energy $E^{(k)}$ required to travel the distance $D = 9$ at $Re = 10^3$ and $Re = 10^4$. All fishes F_1, F_2, F_3 and F_4 present the same tail amplitude $A = 0.4$.*

- **Observations:** Fish F_4 spends least energy

  $\hookrightarrow$ Also slowest $\Rightarrow$ unfair comparison

- **Fair comparaison:** fish with same velocity

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Fish swimming | Power spent

- Same velocity $\Rightarrow$ regulator r of fish tail amplitude $A(c_1, c_2, \ell)$

  $\hookrightarrow$ Target velocity: average velocity of slowest fish (U_4 for F_4)

  $\hookrightarrow$ If $U_i > U_4$ increase A, else if, decrease

$fish$	$Re = 10^3$	$Re = 10^4$
F_1^r	0.64	0.24
F_2^r	0.66	0.26
F_3^r	0.77	0.28
F_4	0.77	0.30

Tab. : *Comparison of the energy $E^{(k)}$ required to travel the distance $d = 9$ at $Re = 10^3$ and $Re = 10^4$. Fishes F_1^r, F_2^r, F_3^r regulated the maximal tail amplitude to swim at the velocity of F_4.*

- **Observations:** Fish F_1 spent least energy,
  Fish F_4 spent most energy.

  $\hookrightarrow$ vertical movements create resistance $\Rightarrow$ least efficient in energy view point

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Fish swimming | Power spent

Gray's paradox [1] :

"the power required for a dolphin of length $1.82m$ *to swim at a speed of* $10.1m/s$ *is about seven times the muscular power available for propulsion (swimming more efficient than rigid body towed at same velocity)*

$\hookrightarrow$ Paradox contested (J. Lighthill [2]) : fish power 3X higher

$\hookrightarrow$ Paradox "confirmed" experimentally at MIT (robot bluefin tuna) by Barret *et al.* [3]

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[1] Gray J. (1936) : Studies in animal locomotion. VI. The propulsive power of the dolphin, *J. Exp. Biol.* **13** pp. 192-199.

[2] Lighthill, M.J. (1971) : Large amplitude elongated-body theory of fish locomotion, *Proc. R. Soc. Mech. B.* **179** pp. 125-138.

[3] Barrett, D.S., Triantafyllou, M.S., Yue, D.K.P., Grosenbauch, M.A., Wolfgang, M.J. (1999) : Drag reduction in fish-like locomotion, *J. Fluid Mech.* **392** pp. 182-212.

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Fish swimming | Power spent

▶**Propulsive index**

$$I_p = \frac{P_{engine}}{P_{ps}}, \quad ps : \text{periodic swim.} \tag{22}$$

$fish$	$Re = 10^3$	$Re = 10^4$
$F1$	0.26	0.31
$F2$	0.26	0.21
$F3$	0.24	0.17
$F4$	0.17	0.14

Tab. : *Propulsive indexes* I_p *evaluated for fishes* F_1, F_2, F_3 *and* F_4 *at* $Re = 10^3$ *and* $Re = 10^4$.

▶ **Observations:** $I_p < 1 \Rightarrow$ power "engine" $<$ power "swim"

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Fish swimming | **Power spent**

▶ **Observation:** swim "costly"

▶ **Idea:** burst and coast swimming

Benefit of gliding periods ?

$\hookrightarrow$ **Definition of Burst and coast :** several cycles

- fish swims from minimal velocity U_i to maximal velocity U_f
- fish glides from maximal velocity U_f to minimal velocityU_i

▷ We choose $U_f = \alpha_f U_{max}$ et $U_i = \alpha_i U_{max}$

▷ **Goal:** Compare burst and coast swimming / periodic swimming (same average velocity)

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Fish swimming | Power spent

Example of burst and coast swimming with $\alpha_i = 0.2$ and $\alpha_f = 0.8$.

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Fish swimming | Power spent

Test case: Fish F_1 at $Re = 10^3$ and at $Re = 10^4$

Efficiency of burst and coast swimming R:

$$R = \frac{P_{bc}}{P_{ps}}, \quad bc : \text{burst and coast.} \tag{23}$$

(α_i, α_f)	$Re = 10^3$	$Re = 10^4$
$(0.2, 0.8)$	0.77	0.85
$(0.6, 0.8)$	1.02	1.00
$(0.4, 0.6)$	0.85	0.81
$(0.2, 0.4)$	0.63	0.71

Tab. : *Efficiency R of burst and coast swimming for fish F_1 at $Re = 10^3$ and $Re = 10^4$ using different couples of $U_f = \alpha_f U_{max}$ and $U_i = \alpha_i U_{max}$.*

$\hookrightarrow$ Burst and coast swimming efficient for low speeds!

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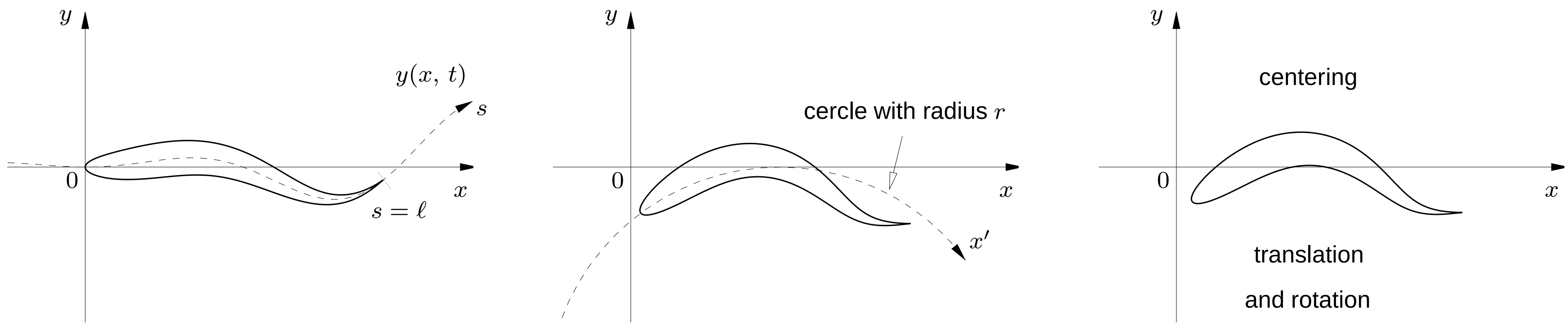
Fish swimming | Maneuvers

Example: predator/prey $\Rightarrow$ reach food

Method: add mean curvature r



(h) Swimming shape (i) Maneuvering shape (j) Real motion shape

Fig. : *Sketch of swimming and maneuvering shape.*

Question: adaptation of $r(t)$?

Fish swimming| Maneuvers

Idea: adapt r using "angle of vision" θ_f, i.e. $r = r(\theta_f)$:

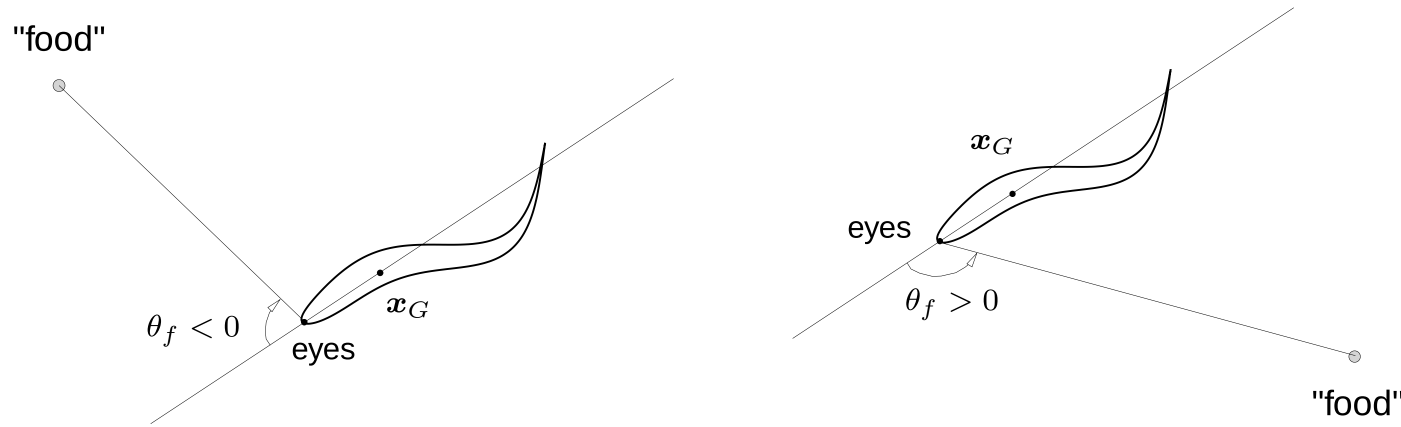


Fig. : *Sketch of the oriented food angle of vision.*

$$r(\theta_f) = \begin{cases} \infty & \text{if } \theta_f = 0, \\ \overline{r} & \text{if } \theta_f \geq \overline{\theta_f}, \\ -\overline{r} & \text{if } \theta_f \leq -\overline{\theta_f}, \\ \overline{r}\left(\frac{\overline{\theta}}{\theta_f}\right)^2 & \text{otherwise.} \end{cases} \qquad (24)$$

We impose $|r| \geq \overline{r}$ and $|\theta_f| \geq \overline{\theta_f}$. We chose arbitrarily $\overline{r} = 0.5$ and $\overline{\theta} = \pi/4$.

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Fish swimming | Maneuvers

$$Re = 10^3$$

Fish swimming | Maneuvers

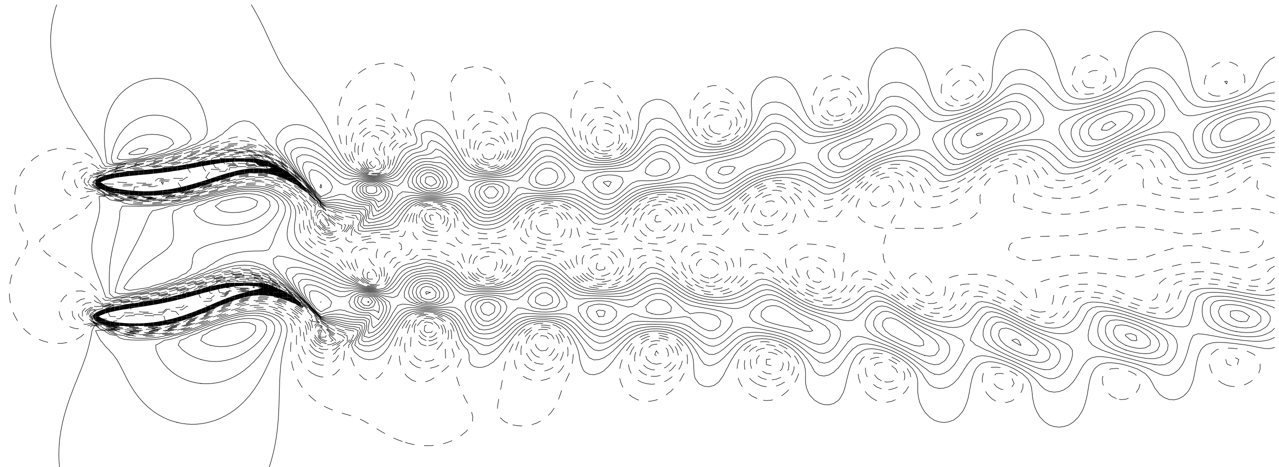
$$Re = 10^4$$

Fish swimming | Schooling

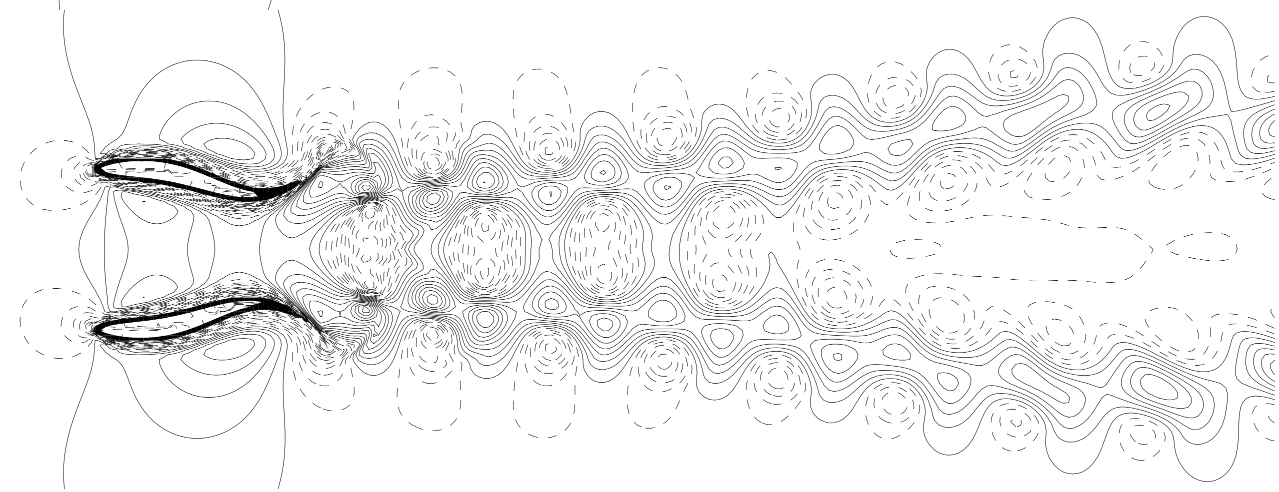
- **Configuration:** school limited to 3 fishes with parameters F_1

$\hookrightarrow$ **Preliminary study** 2 fishes F_1 with parallel swim

Velocity u
Phase



Velocity u
Anti-phase



Fish swimming | Schooling

- **Observation:** existence of a zone in the wake where flow has same velocity sign than fishes velocities
- **Idea:** put a third fish in this zone with "potential benefits"

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Fish swimming | Schooling

▶ **Observation:** existence of a zone in the wake where flow has same velocity sign than fishes velocities

▶ **Idea:** put a third fish in this zone with "potential benifits"

Anti-phase. $\Rightarrow$ Quite efficient.

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Fish swimming | Schooling

▶ **Observation:** existence of a zone in the wake where flow has same velocity sign than fishes velocities

▶ **Idea:** put a third fish in this zone with "potential benifits"

Phase. ⇒ Very efficient.

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Fish swimming | Schooling

- **Goal:** save energy

$\hookrightarrow$ adapt velocity of the third fish

(regulation of tail amplitude A to reach same velocity than two other fishes)

	Phase				Anti-phase			
L D	0.4	0.5	0.6	0.7	0.4	0.5	0.6	0.7
1.5	15.0	16.3	11.1	7.1	6.8	6.9	9.8	7.1
2.0	10.1	14.5	9.8	6.0	6.8	6.1	9.8	6.0
2.5	8.4	13.6	9.0	5.1	6.7	5.3	9.0	5.1
3.0	15.0	15.1	6.9	5.0	5.2	5.1	7.0	3.2
3.5	5.2	13.2	6.2	2.2	4.9	5.0	6.2	0.5

Tab. : *Percentage of energy saved for the three fishes school in comparison with three independent fishes.* $Re = 10^3$.

The 3 fishes school can save an amount around 15% of total energy!!

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Jellyfish swimming

⇒ **Use vortices generated**

Three dimensions | Method

- Study engineering problems : several millions of dofs

  ↪ Required parallel code

  ⇒ One solution: Message Passing Interface (MPI)

  ⇒ Other solution with higher abstraction level:

  Portable, Extensible Toolkit for Scientific Computation (PETSc)

  `http://www.mcs.anl.gov/petsc/petsc-as/`

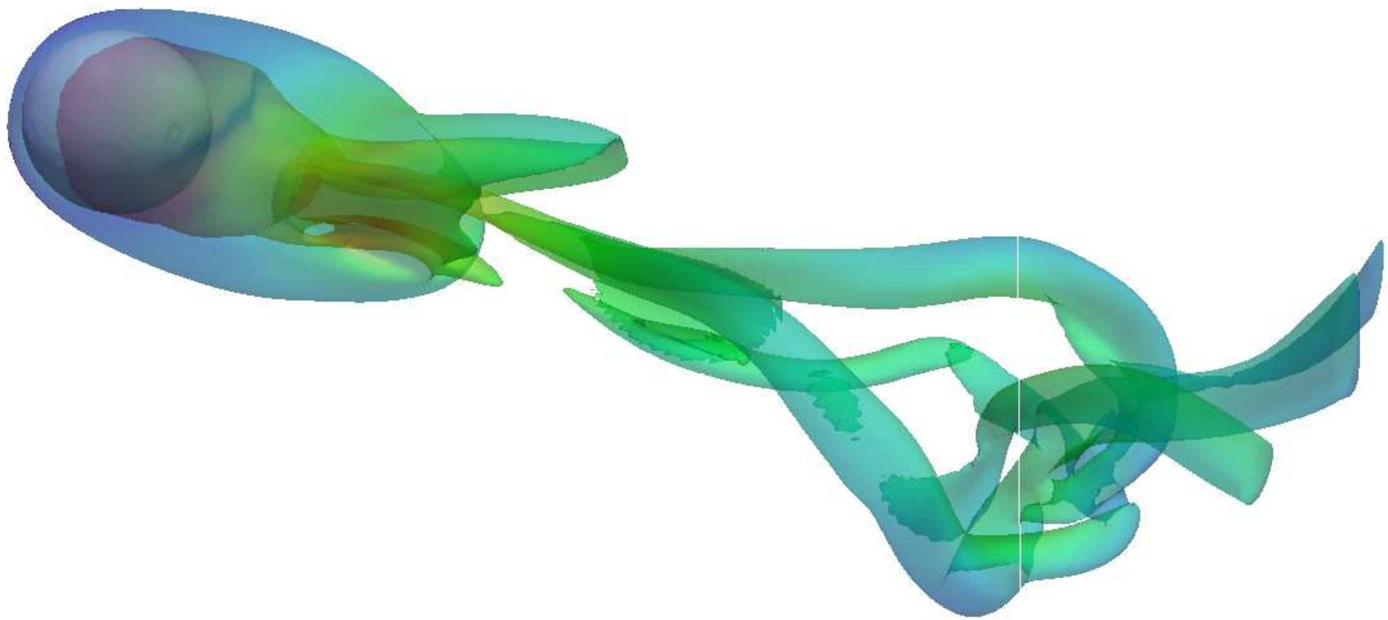
  ↪ PETSc gives:

  – structures for parallelism (DA *Distributed Arrays*),

  – librairies to solve linear systems in parallel (KSP *Krylov Subspace methods*)

Three dimensions | Validation

Sphere at $Re = 500$



$\hookrightarrow C_D = 0.61 \Rightarrow$ in agreement with literature results (and correlations).

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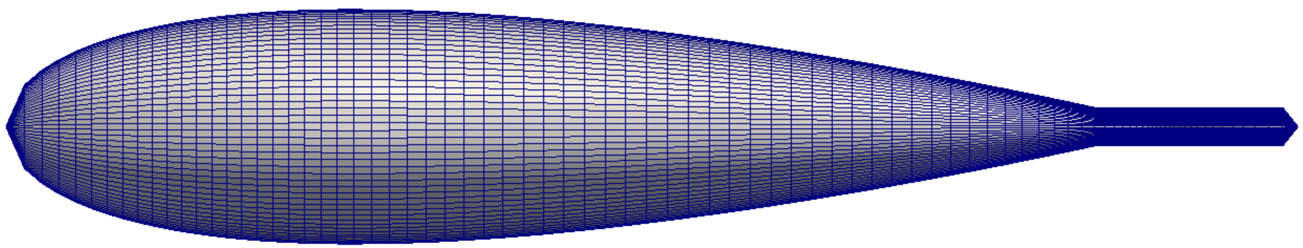
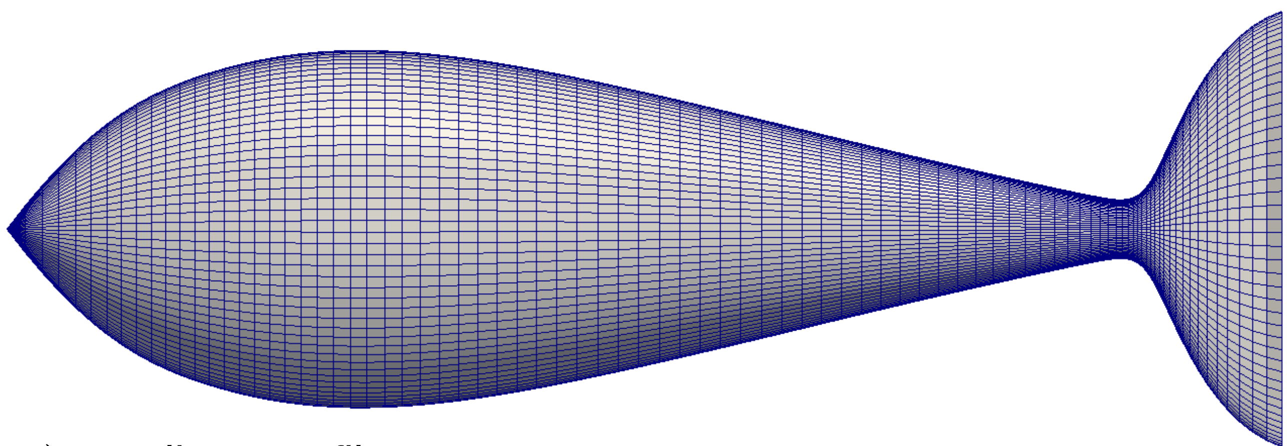
Three dimensions | Fish

- Steady shape: ellipses centered on the backbone x_i, with axis $y(x_i)$ and $z(x_i)$.



$\hookrightarrow$ $y(x_i)$ is NACA0012 profil + tail



$\hookrightarrow$ $z(x_i)$ B-splines profil

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Three dimensions | Fish

- Three dimensions
  - $\hookrightarrow$ periodic, no artificial forces and torques,
  - $\hookrightarrow$ each ellipse is orthogonal to the backbone $\Rightarrow$ mass conservation

Three dimensions | Fish

- Three dimensions
  - $\hookrightarrow$ periodic, no artificial forces and torques,
  - $\hookrightarrow$ each ellipse is orthogonal to the backbone $\Rightarrow$ mass conservation

Three dimensions | Fish

3D fisf $Re = 1000$. *Mesh* $768 \times 128 \times 256$

$\Rightarrow$ 3D and 2D wakes behavior are different

`S.Kern and P. Koumoutsakos, J Exp. Biology` **`209`**`, 2006.`

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Three dimensions | Fish maneuvers

3D fisf $Re = 1000$. *Mesh* $512 \times 128 \times 512$

$\Rightarrow$ Turn seems more difficult than in 2D case ...

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Three dimensions | Fish maneuvers

3D fisf $Re = 1000$. *Mesh* $512 \times 128 \times 512$

$\Rightarrow$ Quasi 2D (fish height is constant $y = 0.3$) $\Rightarrow$ more efficient

Three dimensions | Fish schooling

3D fisfes $Re = 1000$. *Mesh* $768 \times 128 \times 256$

$\Rightarrow$ No efficient effect for 3^{rd} fish. 3D wake $\neq$ 2D wake (no inverted VK street)

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Three dimensions | Jellyfish

3D jellyfish $Re = 1000$. *Mesh* $256 \times 256 \times 512$.

$\Rightarrow$ Velocity very close to 2D case (quasi axi-symetric)

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Conclusions

METHODS

- **Cartesian meshes and penalization**
  - ▷ **Advantages:** simple numerical algo. and parallelism
  - ▷ **Drawbacks:** precision, turbulence, boundary layers
    - $\hookrightarrow$ **Solution:** local refinement "octree" or global multi-grids, improve penalization order (2^{nd} order), ... (?)

- **Collocation scheme:** non oscillating compact schemes
  - ▷ **Advantages:** only one grid (parallelism), simple boundary conditions
  - ▷ **Drawbacks:** no spurious modes but discrete conservations not exactly satisfied
    - $\hookrightarrow$ **Solution:** 4^{th} order correction (E. Dormy, JCP **1**51), MAC, ..

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Conclusions

RESULTS

- **Dimension 2**
  - ▷ Validation test case cylindre
  - ▷ Self propelled fishes
    - ↪ Modeling BCF (tuna, eels, etc..)
    - ↪ Energetic study
    - ↪ Maneuvers, turns
    - ↪ Fish schooling efficient
- **Dimension 3** (now and future...)
  - ▷ Validation sphere
  - ▷ Self propelled fishes
  - ▷ Jellyfish

$\Rightarrow$ **Validations and improvement are still necessary**

Next ....

- **Fluid-Structure interactions & elasticity** (eulerian, post doc Thomas Milcent)
  - $\hookrightarrow$ Model the tail/fins
  - $\rightarrow$ Example: cylinder motion imposed by penalization with free motion of the "tail"

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